



פרויקט מס' BS-CS-26-010

Stability of Performance of Leveraged ETFs

שיערוך הביצועים של תעודות מינוף

פרויקט מחקרי

דוח מסכם

הוכן לשם השלמת הדרישות לקבלת

תואר ראשון במדעי המחשב B. Sc.

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הוגש למחלקה למדעי המחשב

המכללה האקדמית להנדסה סמי שמעון

באר-שבע

תאריך לועזי

31/05/2026

תאריך עברי

ט"ז בסיוון ה'תשפ"ו



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Abstract

Leveraged Exchange-Traded Funds (LETFS) are financial instruments designed to deliver a multiple, typically a double or triple of the daily performance of an underlying benchmark. These instruments have become increasingly popular among retail investors seeking amplified exposure. Their mechanics however, can produce unexpected behavior and results that are not seen in regular ETFs.

To maintain a constant leverage ratio, an LETF must reset its exposure at the close of every trading day. As a result the fund compounds from a base that shifts each day, and its return over a long timeframe depends not only on where the benchmark ends up, but on the path it took to get there.

This note provides a framework to better understand how the entry point and timing affects the outcomes and how its compared to the underlying benchmarks.

The framework includes hypothetical funds launched at different points in time and compare their trajectories over a common evaluation window.

תקציר

קרנות סל ממונפות הן מכשירים פיננסיים שנועדו לספק מכפיל, בדרך כלל כפול או משולש, מהביצועים היומיים של מדד ייחוס. מכשירים אלה הפכו פופולריים יותר ויותר בקרב משקיעים קמעונאיים המחפשים חשיפה מוגברת. עם זאת, המכניקה שלהם יכולה לייצר התנהגות ותוצאות בלתי צפויות שלא נראות בקרנות סל רגילות.

כדי לשמור על יחס מינוף קבוע, תעודת מינוף חייבת לאפס את החשיפה שלה בסוף כל יום מסחר. כתוצאה מכך, הקרן מורכבת מבסיס שמשתנה בכל יום, והתשואה שלה לאורך זמן ארוך תלויה לא רק במקום שבו מדד הייחוס מסתיים, אלא גם בנתיב שעברה כדי להגיע לשם.

המחקר מספק מסגרת להבנה טובה יותר כיצד נקודת הכניסה והתזמון משפיעים על התוצאות וכיצד הם בהשוואה למדד הייחוס הבסיסי.

המסגרת כוללת קרנות היפותטיות שהושקו בנקודות זמן שונות ומשווה את מסלוליהן על פני חלון הערכה משותף.

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1. Introduction

Introduced in 2006, leveraged ETFs are financial instruments designed to deliver a multiple of the daily performance of an underlying index or asset. An LETF applies derivatives to magnify the exposure to a particular index or other targeted asset, most do not aim to amplify the monthly or annual returns of the target assets but instead track the daily changes. This resetting effect allows the ETF to seek 3x or 2x leverage daily but can cause longer-term returns to diverge significantly from simply compounding the underlying index's returns.

Leverage ETFs are path dependent through daily rebalancing mechanics, which fundamentally changes the proposition. The volatility drag from leverage compounding means that even with correct directional conviction, the sequence of returns matters more than the cumulative return. For example, imagine a 2x leveraged ETF tracking an index starting at 100\$.

Day 1: Index drops 10% (100\$ -> 90\$). The 2x ETF drops 20% (100\$ -> 80\$).

Day 2: Index gains 11.1% (90\$ -> 100\$). The 2x ETF gains 22.2% (80\$ -> 97.6\$).

The result is the index is back at 100\$, but the ETF is at 97.6\$, the ETF share has lost value despite the index breaking even because of the volatile path. The daily rebalancing causes the returns compounded over any holding period longer than one day to differ from the promised ratio. As a result, most investors will avoid these tracking errors by limiting the length of their holding periods. In addition, the embedded leverage makes these ETFs risky and speculative in nature.

Consequently, they are used by short-term traders who want to express their directional views on the underlying benchmarks or to implement certain trading strategies.



1.1. Mechanisms of LETFs

1.1.1. Derivatives

A derivative is a financial product whose value is based upon that of another asset. For example, stock options are derivative securities because their value is based on the share price of a publicly traded company, such as General Electric (GE). Options provide their owners with the right, but not the obligation, to purchase or sell shares of a stock at a specific price by a specific date. The investor makes or loses money on the difference between the options price and the actual price on that date. The value of an option, therefore, is derived from a stock's share price but does not reflect actual ownership of those shares.

Leveraged ETFs aim to provide returns that are a multiple of the underlying index. For example, the ProShares Ultra S&P 500 ETF seeks to provide investors with returns that equal twice the performance of the S&P 500 index. If the S&P 500 index rises 1% during a trading day, shares of the ProShares Ultra S&P 500 ETF would be expected to climb 2%.

This type of ETF can be considered a derivative-based ETF because the assets in its portfolio are themselves derivative securities.

1.1.2. Leverage

In leveraged ETFs, leverage is the use of financial derivatives (such as swaps, futures, and options) to multiply the daily returns of an underlying index.

While a standard ETF tracks an index on a 1:1 basis, a leveraged ETF uses debt and derivatives to amplify that movement by 2x or 3x.

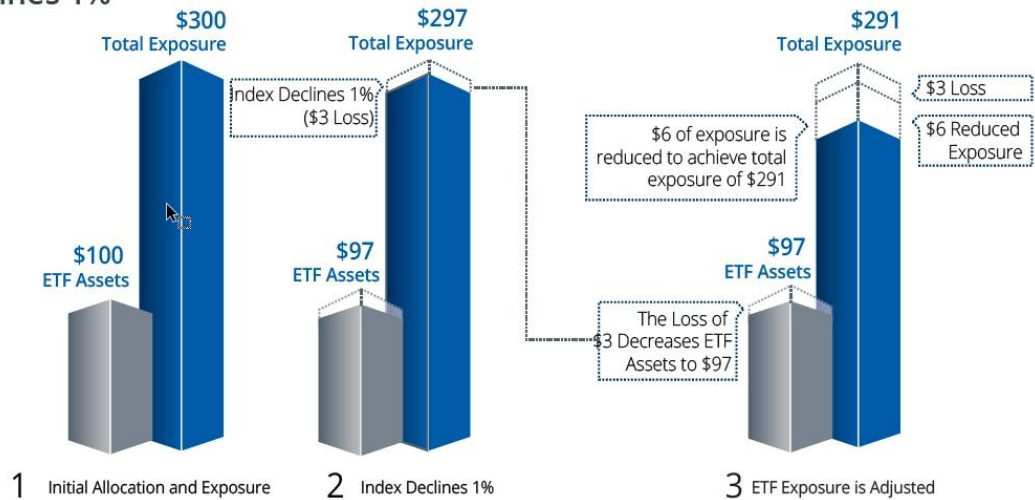
Scenario	Index Return(1x)	LETF Return(3x)
Market Rises	Gains +2%	Gains +6%
Market Falls	Loses -3%	Loses -9%

1.1.3. Daily Rebalancing

Hypothetical Example (A Daily 3X Bull Fund)

Index Declines 1%

(in millions)



1. INITIAL ALLOCATION AND EXPOSURE:

If a 3X ETF has \$100 million in net assets, \$300 million of net exposure to the fund's underlying index must be maintained.

2. INDEX DECLINES 1%

If the index decreases by 1% in a trading day, the gross exposure would decline to \$297 million and net assets would decline to \$97 million, resulting in a \$3 million loss.

3. ETF EXPOSURE ADJUSTED

Since 300% of \$97 million equals \$291 in exposure, the current exposure must be reduced by \$6 million from \$297 million to \$291.

To maintain a fixed leverage ratio, the fund must rebalance its exposure at the end of each trading day. After a price move, the ratio of exposure to net assets drifts away from the target, so the fund adjusts its derivative/swap positions. Critically, this requires buying exposure after gains and selling after losses, this happens mechanically.



2. Literature Review

2.1. Background

2.1.1. Leveraged ETFs

Leveraged exchange-traded funds (LETFs) are relative newcomers to the world of exchange-traded funds (ETFs). An LETF tracks the value of an index, a basket of stocks, or another ETF, with the additional feature that it uses leverage. For instance, the ProShares Ultra Financial ETF (UYG) offers double exposure to the Dow Jones U.S. Financials index. To achieve this, the manager invests two dollars in a basket of stocks tracking the index per each dollar of UYG's net asset value, borrowing an additional dollar. This is an example of a long LETF. Short LETFs, such as the ProShares UltraShort Financial ETF (SKF), offer a negative multiple of the return of the underlying ETF. In this case, the manager sells short a basket of stocks tracking the Dow Jones U.S. Financials index (or equivalent securities) to achieve a short exposure in the index of two dollars per each dollar of NAV ($\beta = -2$). In both cases, the fund's holdings are rebalanced daily [2].

It has been empirically established that if we consider investments over extended periods of time (e.g, three months, one year, or more), there are significant discrepancies between LETF returns and the returns of the corresponding leveraged buy-and-hold portfolios composed of index ETFs and cash. Since early 2008, the quarterly performance of LETFs over any period of 60 business days has been inferior to the performance of the corresponding static leveraged portfolios for many leveraged/unleveraged pairs tracking the same index. There are a few periods where LETFs actually outperform, so this is not just a one-sided effect [2].



2.1.2. Performance Evaluation

Variance is most useful in determining the overall spread of a data set. Range simply takes two data points, the lowest and highest values, and measures the space between them. The interquartile range also interprets the amount of spread within a set of data by using only the middle 50% of scores. Unlike range, variance takes into account every data point within the set and measures each distance from the mean. Variance is calculated using a number of values from data and the associated distribution. Specifically, variance requires raw data scores and the sample size of the data.

The formula for population variance can be defined as:

$$\sigma^2 = \frac{\sum (X - \mu)^2}{N}$$

where σ^2 represents the population variance, $X - \mu$ represents the deviation between each score and the population mean, and N is the population size [3].

Geometric mean of a data set $\{a_1, a_2, \dots, a_n\}$ is given by [4]

$$\left(\prod_{i=1}^n a_i \right)^{\frac{1}{n}} = \sqrt[n]{a_1 a_2 \dots a_n}$$

the geometric mean is useful whenever the quantities to be averaged combine multiplicatively, such as population growth rates or interest rates of a financial investment.

Return on investment used to evaluate the efficiency or profitability of an investment or to compare the efficiency of a number of investments.

Calculation: compares the annual net benefits of an investment to the cost of the investment; expressed as a percentage.

$$\text{ROI (\%)} = \frac{\text{Net benefits}}{\text{Costs}} \times 100 \text{ [5].}$$

2.1.3. Technical Tools

This project relies on several Python libraries, each handling a specific part of the workflow:

Matplotlib is a portable 2D plotting and imaging package aimed primarily at visualization of scientific, engineering, and financial data [6].

Yfinance package provides easy access to Yahoo! Finance's API to retrieve market data. It includes classes and functions for downloading historical market data, accessing ticker information, managing cache, and more [7].

NumPy is an open-source library available in Python, which helps in mathematical, scientific, engineering, and data science programming. It is a very useful library to perform mathematical and statistical operations in Python [8].

Pandas is one of the most widely used Python libraries in data science and analytics. Pandas is an open source Python library for data analysis. It gives Python the ability to work with spreadsheet-like data for fast data loading, manipulating, aligning, merging etc [9].



2.2. Related Work

2.2.1. Compounding Dynamics in Leveraged ETFs

The mathematical behaviour of leveraged exchange-traded funds has been studied extensively since these products became widely available, with the majority of the literature converging on a single central mechanism: the daily rebalancing requirement. Because an LETF must reset its exposure at the end of every trading day to maintain a constant leverage ratio, its multi-period return is a product of daily leveraged returns rather than a leveraged multiple of the cumulative benchmark return. Below we review the principal studies that have characterised this mechanism and quantified its consequences.

Long-term tracking behaviour of leveraged indexes. A foundational strand of the literature examines how far LETF returns drift from their stated multiple as the holding horizon extends. Research on long-term return estimation of leveraged indexes and ETFs demonstrates that over horizons no longer than approximately one month, $2\times$ and $-2\times$ daily leveraged products deliver close to their target multiple of the underlying benchmark's return. For spans exceeding one month, however, serious and systematic deviations emerge, and these deviations compound with time [10].

2.2.2. Holding Periods and Outcomes

A second and closely related body of research shifts focus from the mechanism itself to its practical consequences for investors, specifically addressing the question of how long a leveraged position can reasonably be held.

Decay thresholds and optimal holding periods. A study on forecasting holding periods for leveraged ETFs introduces the concept of a decay threshold a maximum level of compounding-induced value erosion that an investor is willing to tolerate and derives the corresponding maximum holding period as a function of the leverage ratio, the elapsed time, the index return, and the prevailing volatility level. The authors demonstrate that the conventional advice to hold leveraged products for no more than a single day is unnecessarily restrictive: for a decay threshold of only -2% and under low-volatility conditions, holding periods extending beyond six months can be justified for certain leveraged funds [11].

Investment shortfalls across holding periods. Complementary work on leveraged and inverse ETFs, holding periods, and investment shortfalls quantifies how the combination of holding period length and entry date jointly determines the risk of an investor failing to achieve the return implied by the product's stated multiple [12].

3. Methods

This note provides an empirical analysis of entry timing effects across 18 LETFs (5 of them are 2x leveraged and the rest are 3x leveraged), grouped by benchmark and sector, with full statistical characterization of the daily return distributions encountered under each entry window. The analysis spans nine entry dates between July 2017 and July 2025, with all portfolios valued as of July 2026. We additionally examine fixed one-year holding periods to isolate the effect of a single market cycle on leveraged outcomes.

3.1. Data

Daily adjusted closing prices for 16 benchmark ETFs were downloaded from Yahoo Finance covering July 2016 to July 2026 (ten years, providing enough market data). All 18 LETF simulations are derived from the historical LETF market data. The following table contains all the LETF tickers and their respective benchmarks:
(The table is in the next page).



LETF	Benchmark	Benchmark Tracks	Leverage	Sector
TQQQ	QQQ	Invesco QQQ Trust (NASDAQ-100)	3x	Technology
QLD	QQQ	Invesco QQQ Trust (NASDAQ-100)	2x	Technology
SSO	SPY	SPDR S&P 500 ETF Trust	2x	Broad Market
UPRO	SPY	SPDR S&P 500 ETF Trust	3x	Broad Market
TECL	XLK	Technology Select Sector SPDR	3x	Technology
SOXL	SOXX	iShares Semiconductor ETF	3x	Semiconductor
FAS	IWB	iShares Russell 1000 ETF	3x	Finance
NAIL	ITB	iShares U.S. Home Construction ETF	3x	Real Estate
DUSL	XLI	Industrial Select Sector SPDR	3x	Industrial
ERX	XLE	Energy Select Sector SPDR	2x	Energy
DRN	XLRE	Real Estate Select Sector SPDR	3x	Real Estate
CWEB	KWEB	KraneShares CSI China Internet ETF	2x	Internet
GUSH	XOP	SPDR S&P Oil & Gas Exploration & Production	2x	Energy
LABU	XBI	SPDR S&P Biotech ETF	3x	Biotechnology
EDC	EEM	iShares MSCI Emerging Markets ETF	3x	Emerging Markets
YINN	JK8.SI	FTSE China 50 exposure (Singapore-listed proxy)	3x	China Equity
DPST	KRE	SPDR S&P Regional Banking ETF	3x	Regional Banking
TMF	TLT	iShares 20+ Year Treasury Bond ETF	3x	Government Bonds

3.2. Simulating Entries

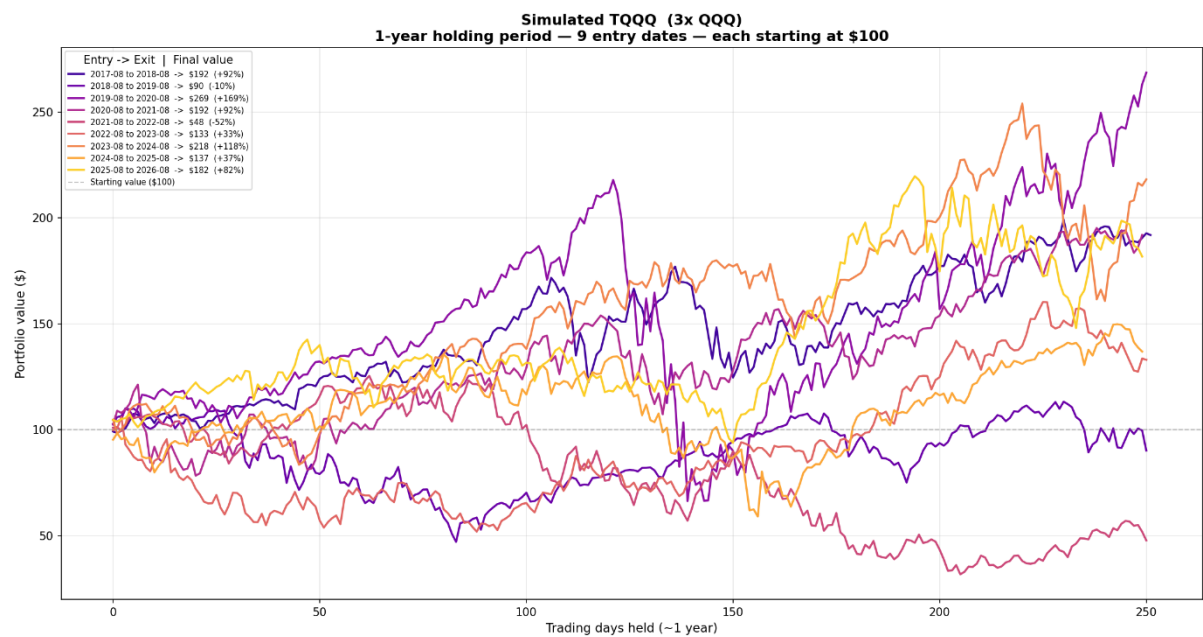
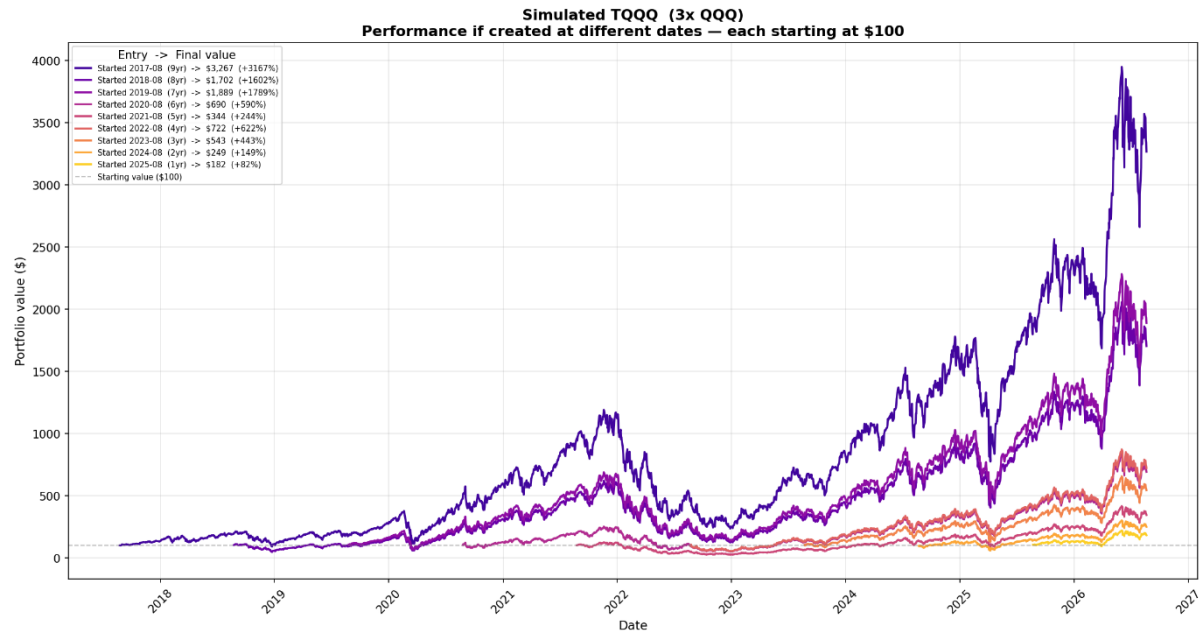
Nine entry dates are tested for each ticker: July of each year from 2017 to 2025. Two holding period configurations are analyzed:

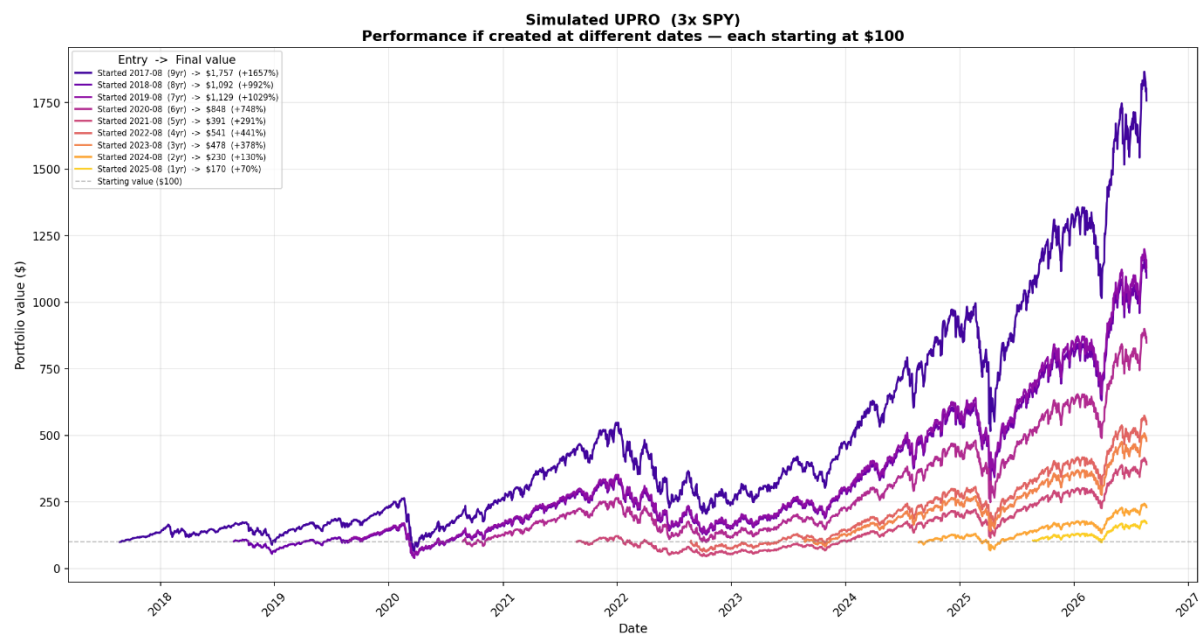
- **Variable holding:** each entry runs from the entry date to July 2026, capturing 1 to 9 years of compounded returns.
- **Fixed 1-year holding:** each entry runs for exactly one calendar year (or to July 2026 if the window would exceed it), placing all lines on a common trading-day x-axis for direct comparison.

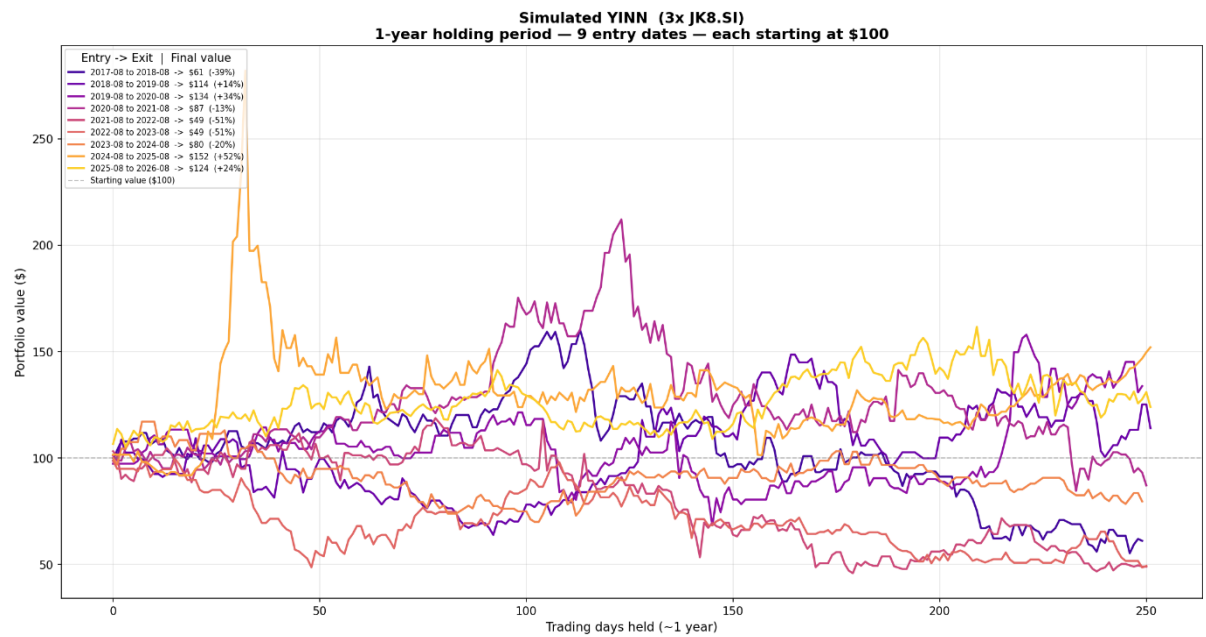
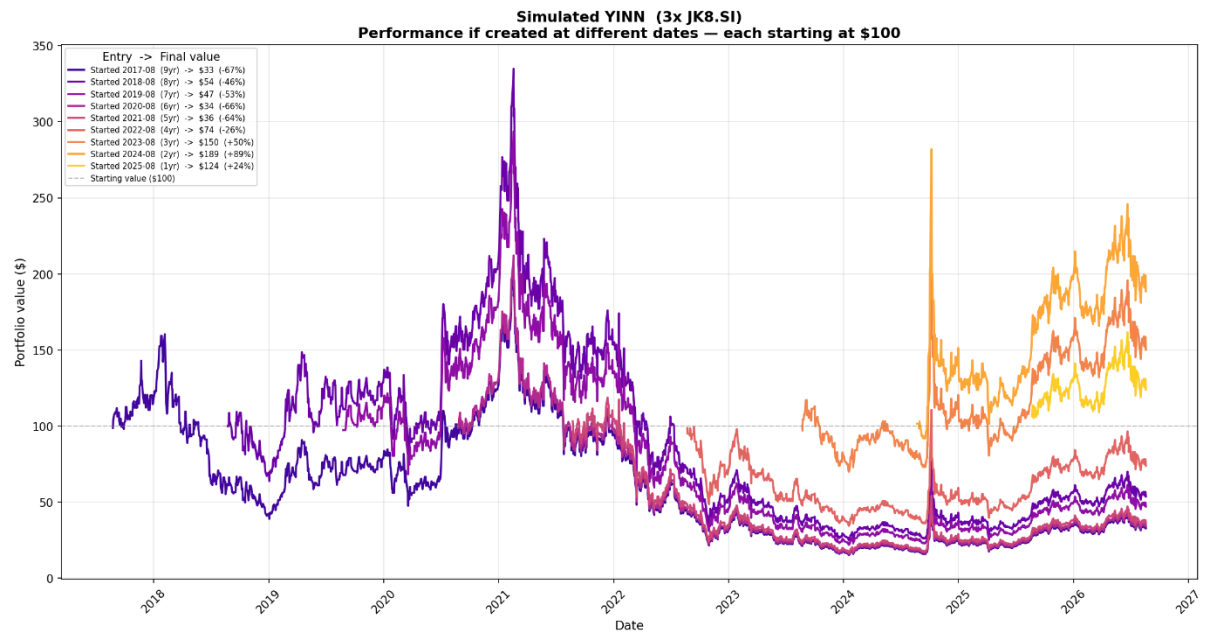
All portfolios are initialised at \$100.

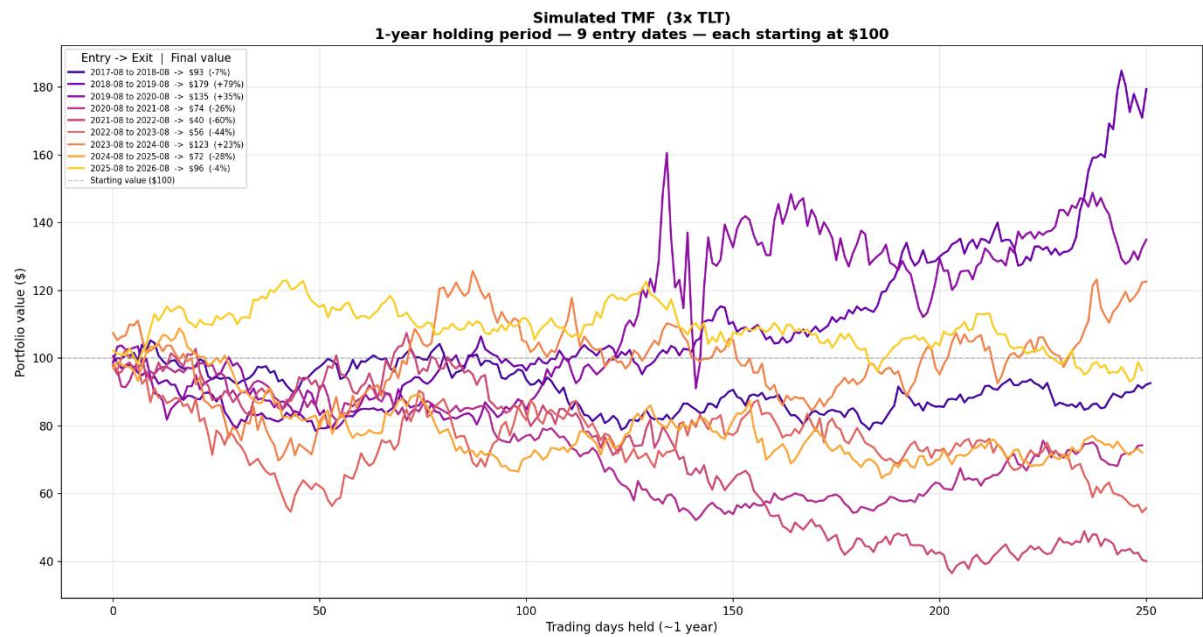
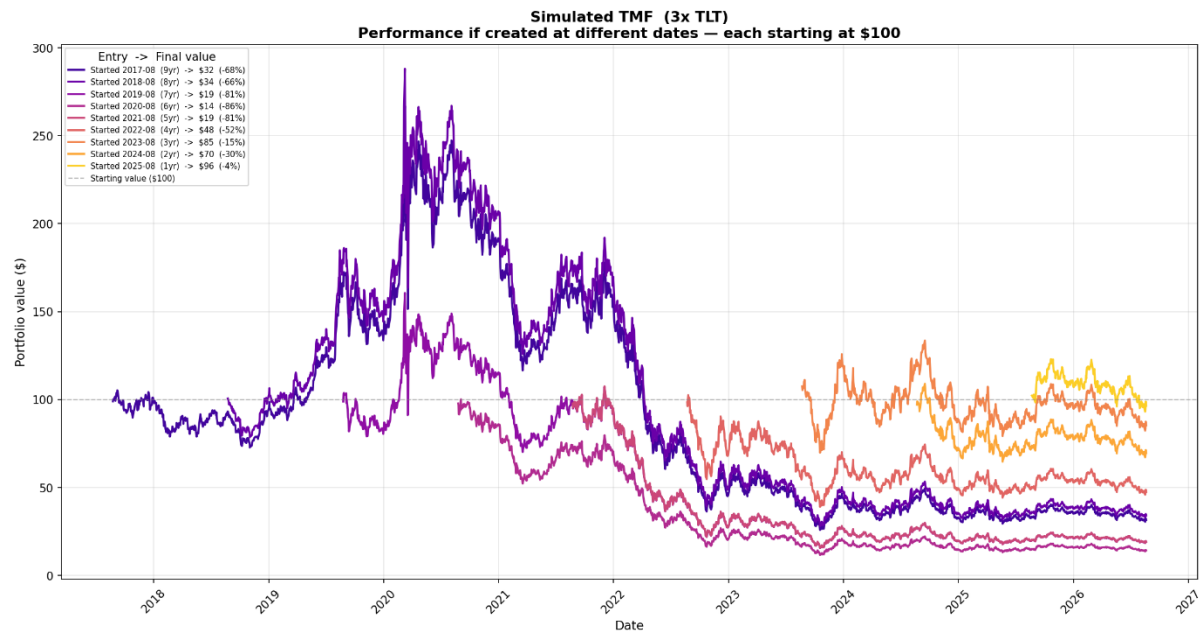
4. Results and Discussion

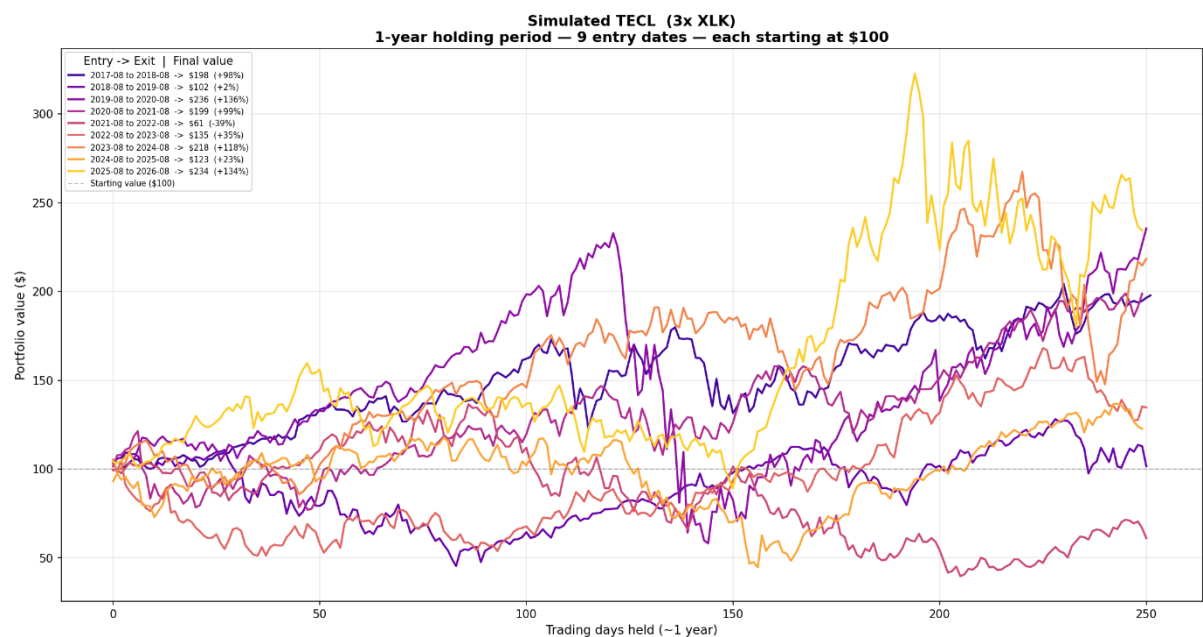
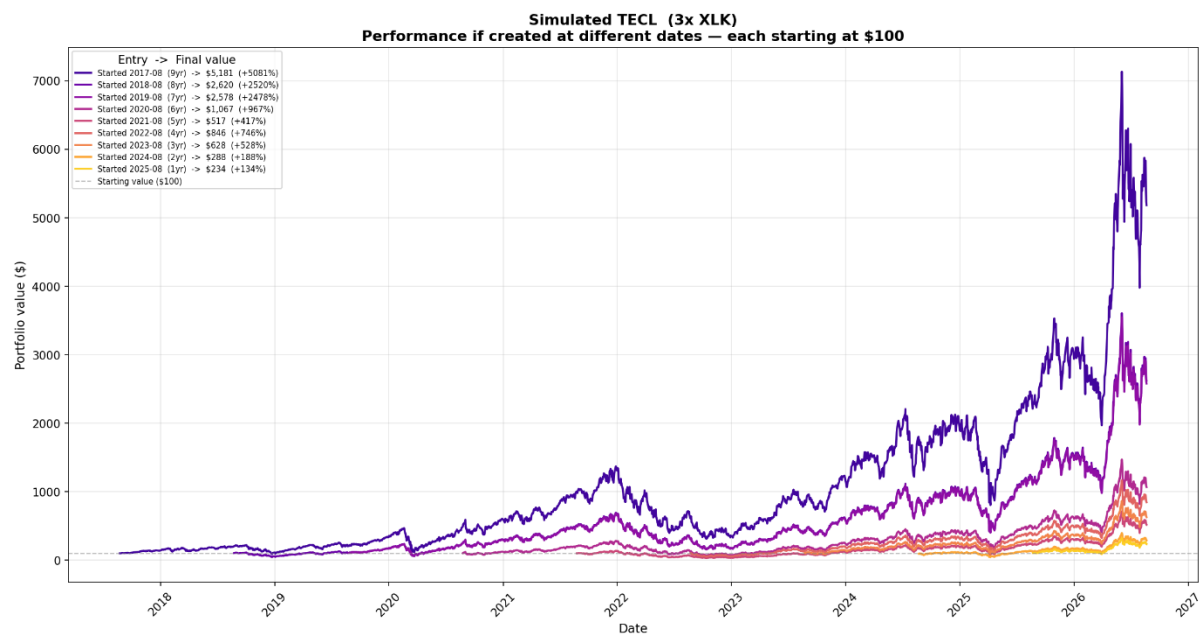
4.1. Charts

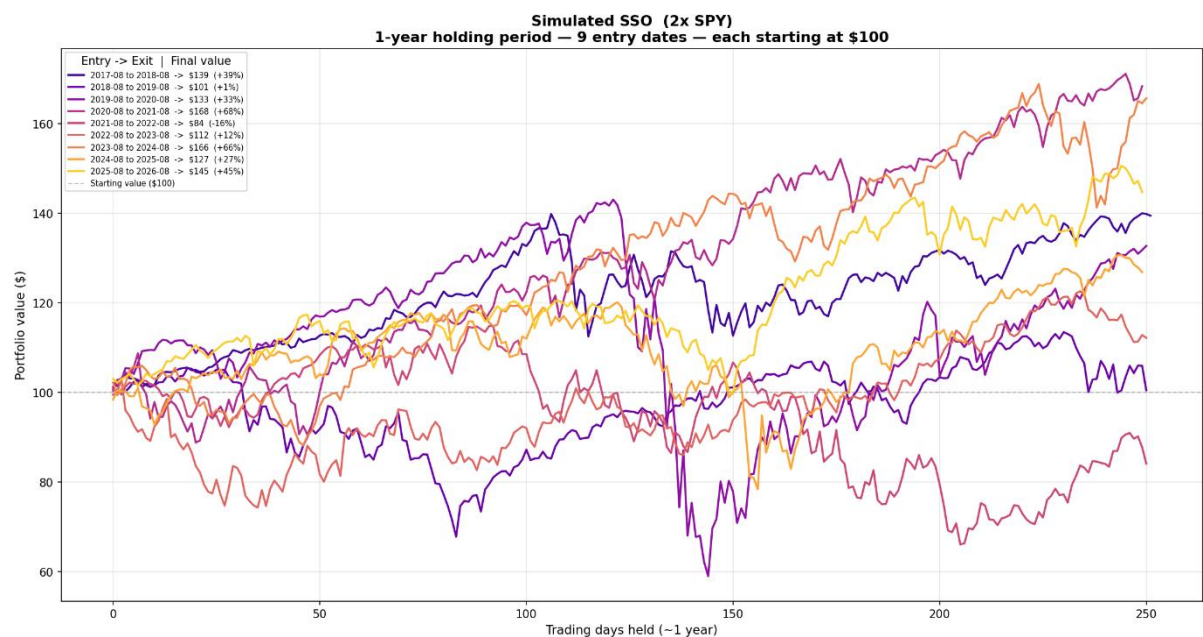
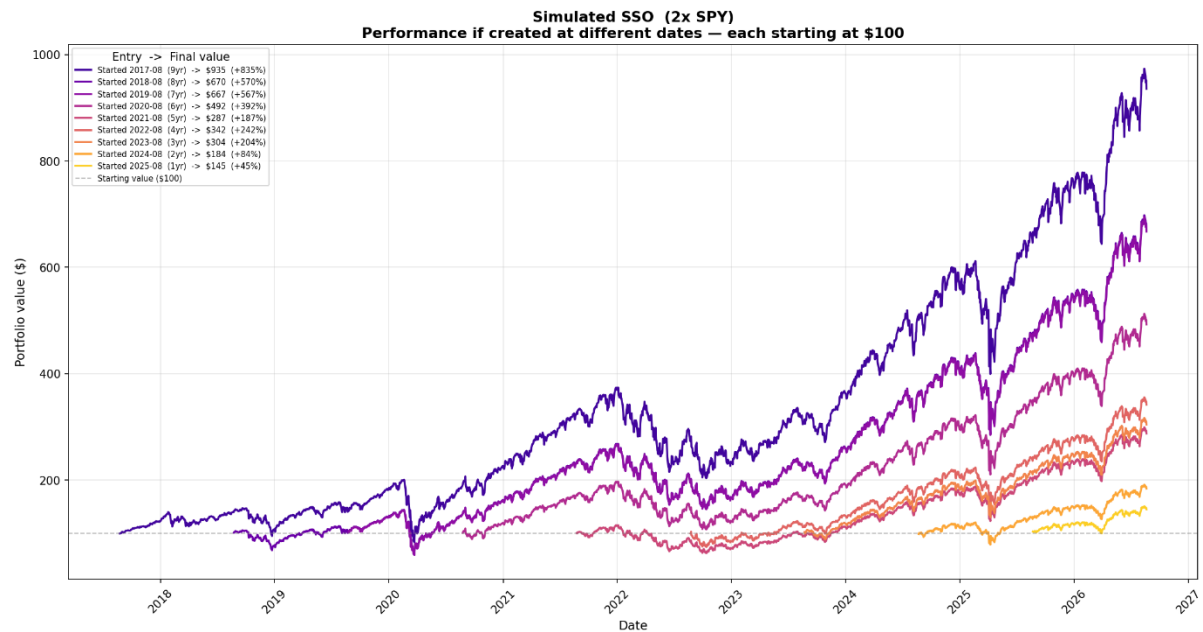


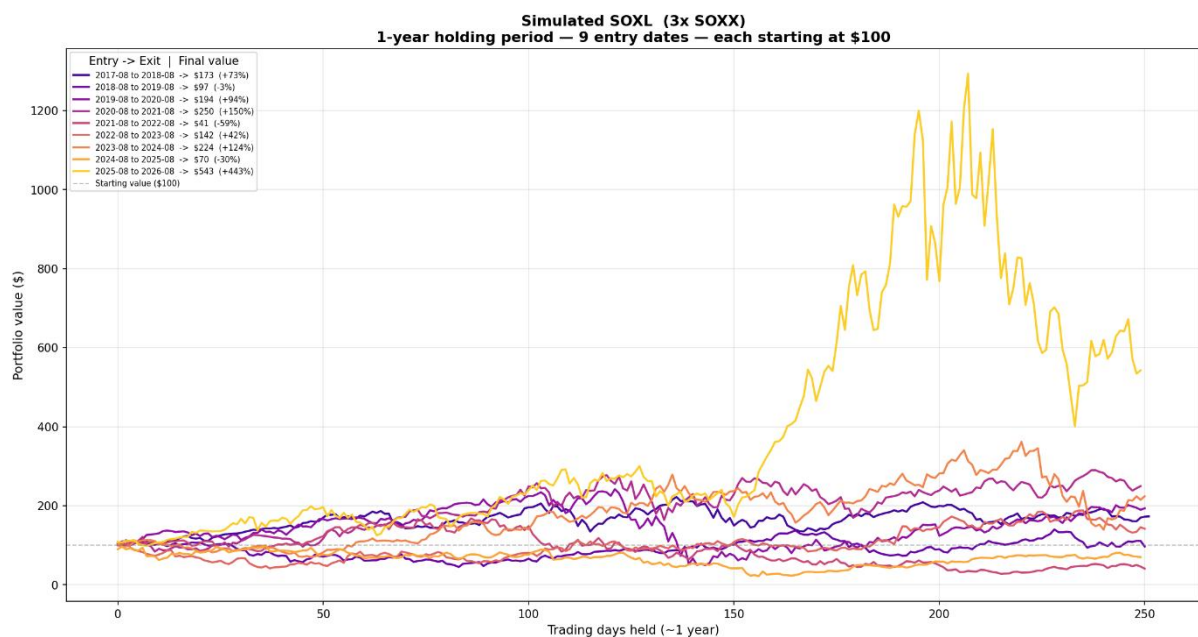










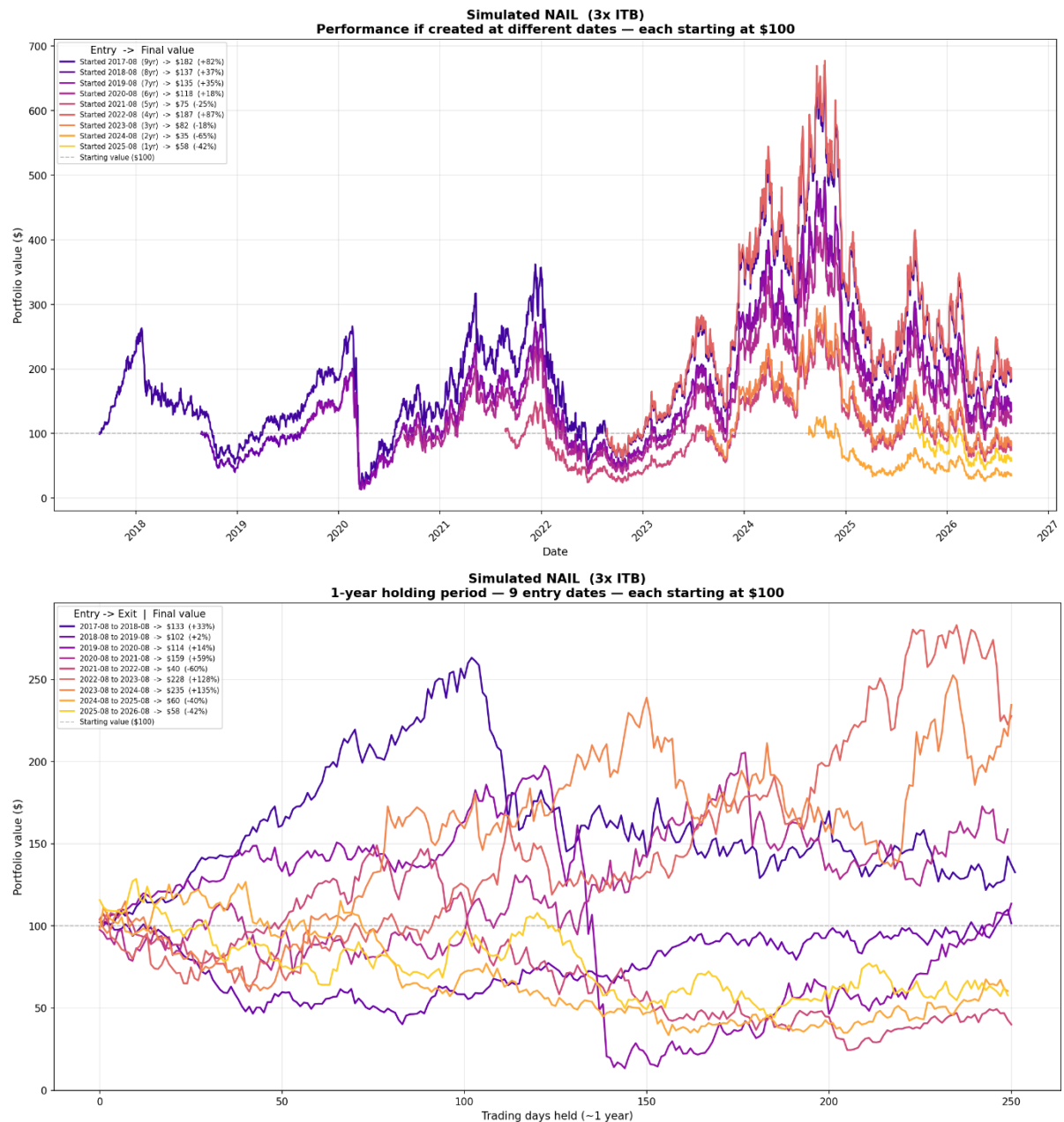


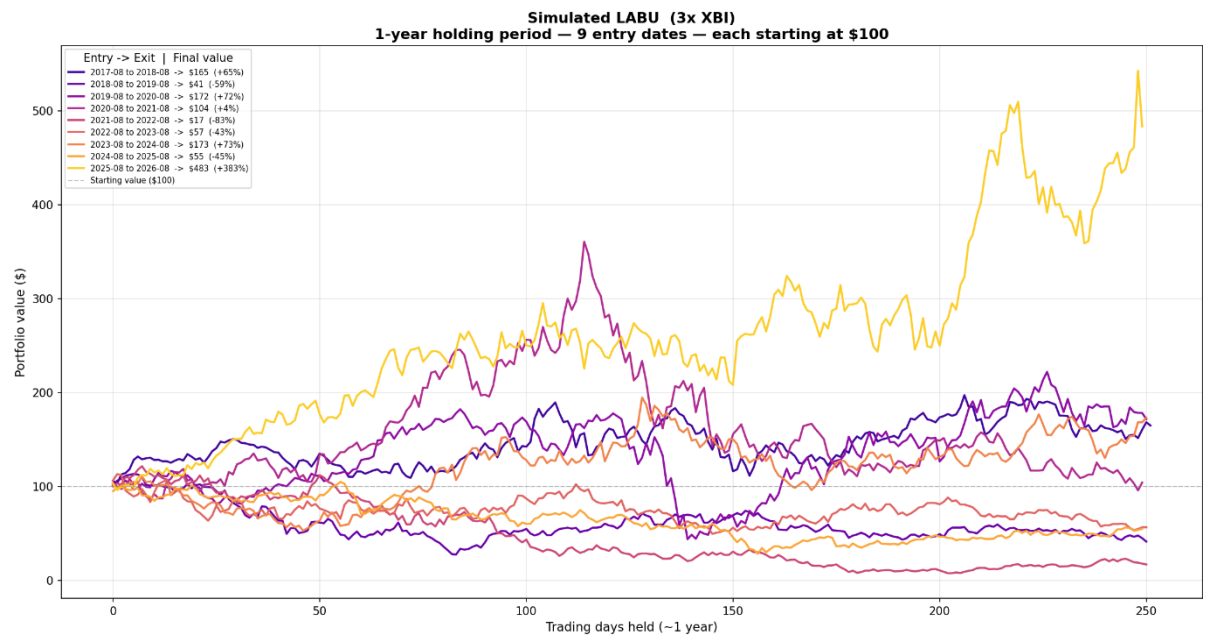
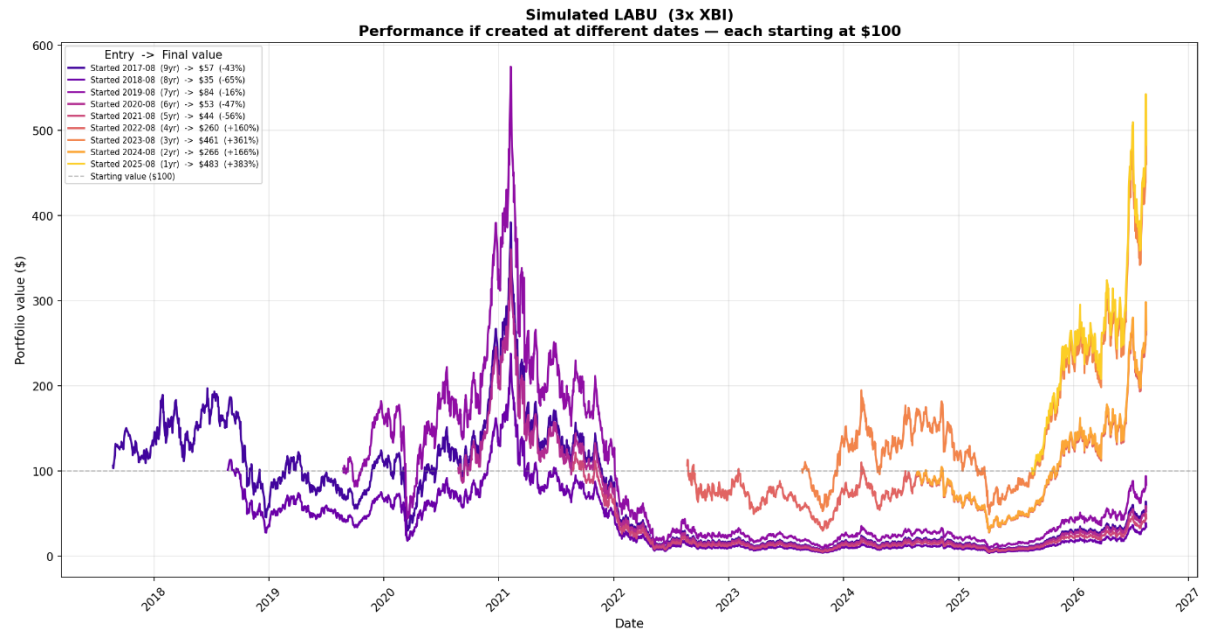
Simulated QLD (2x QQQ)
Performance if created at different dates — each starting at \$100

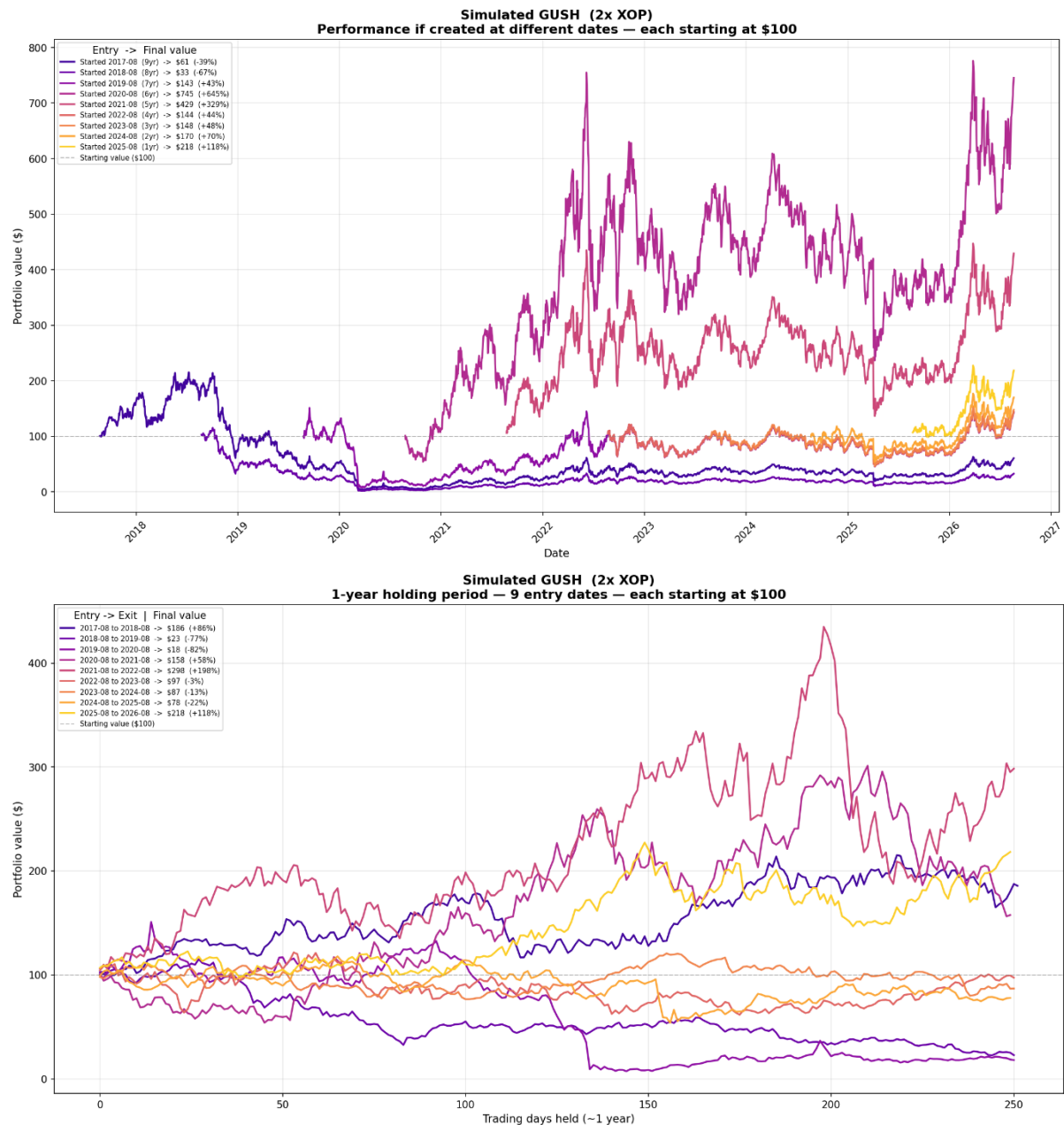


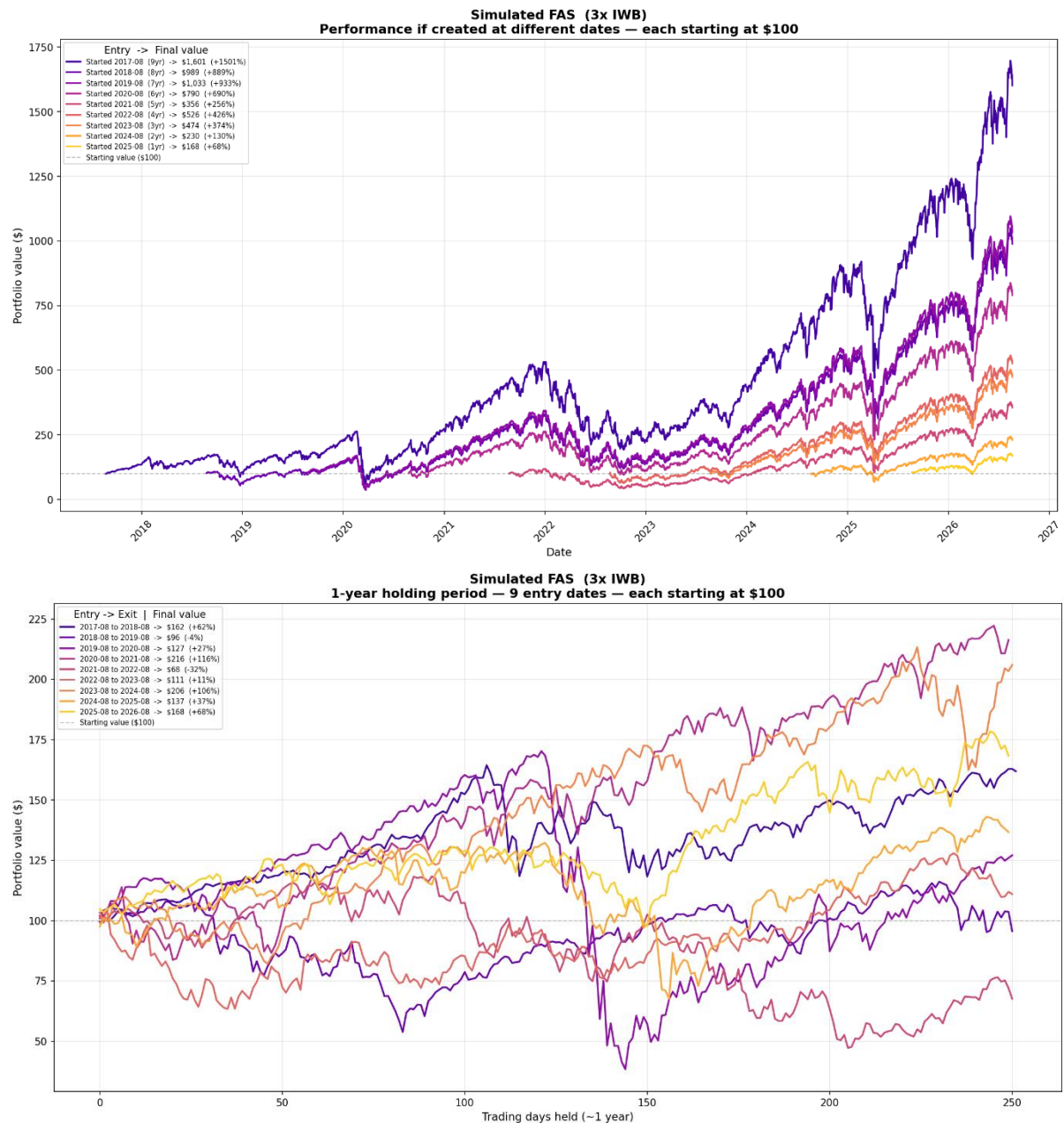
Simulated QLD (2x QQQ)
1-year holding period — 9 entry dates — each starting at \$100

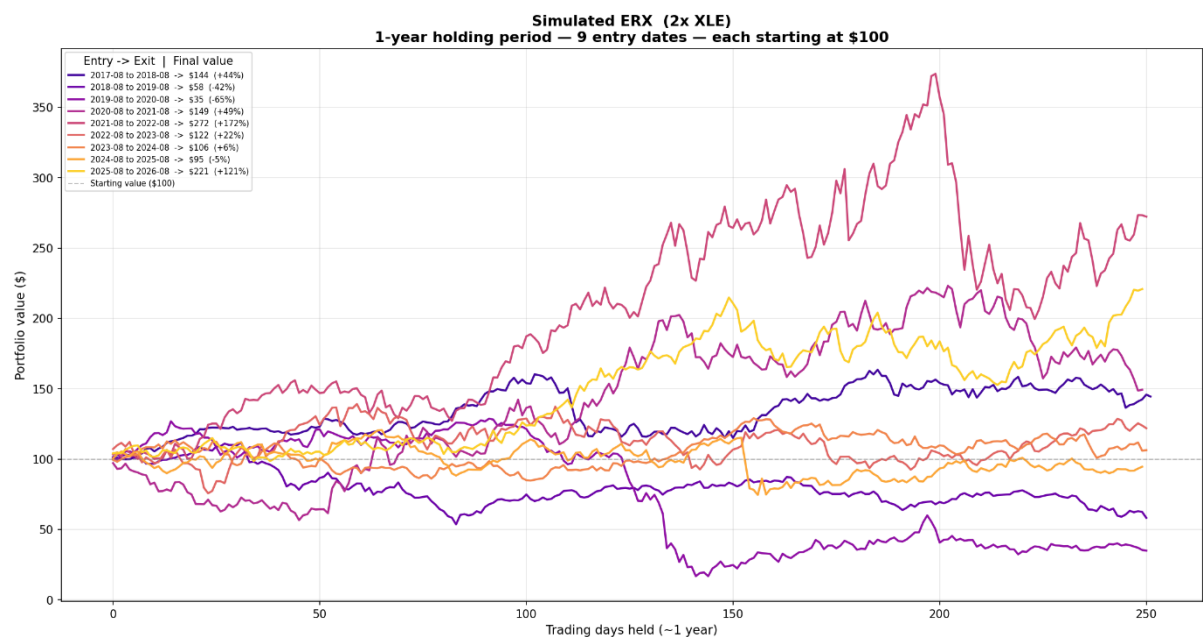
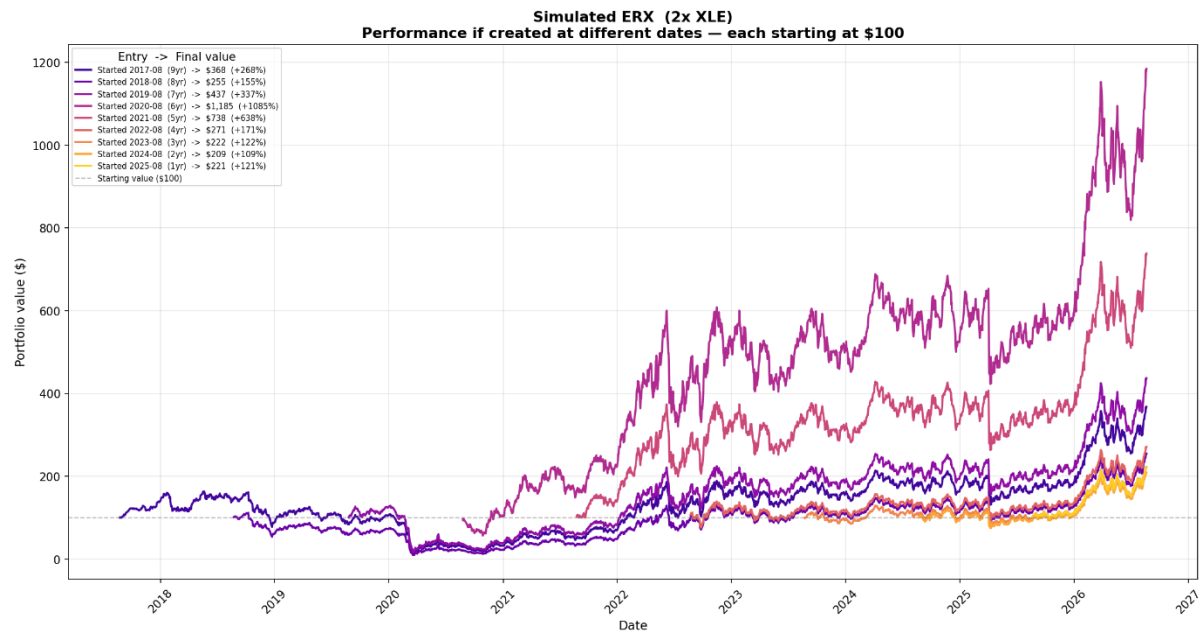


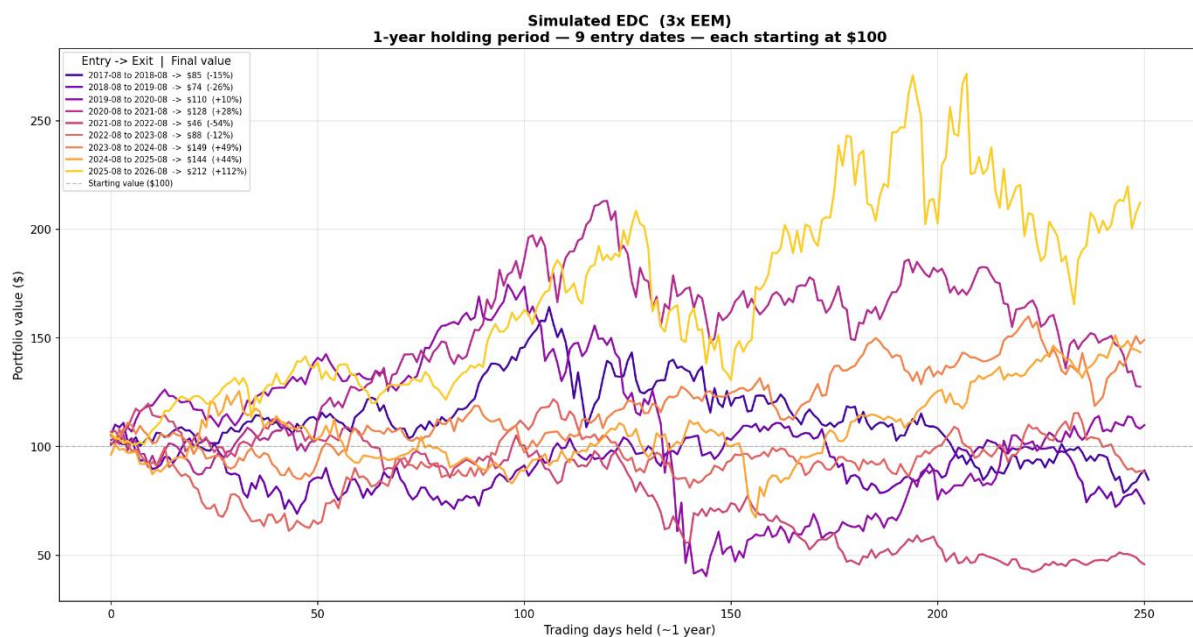
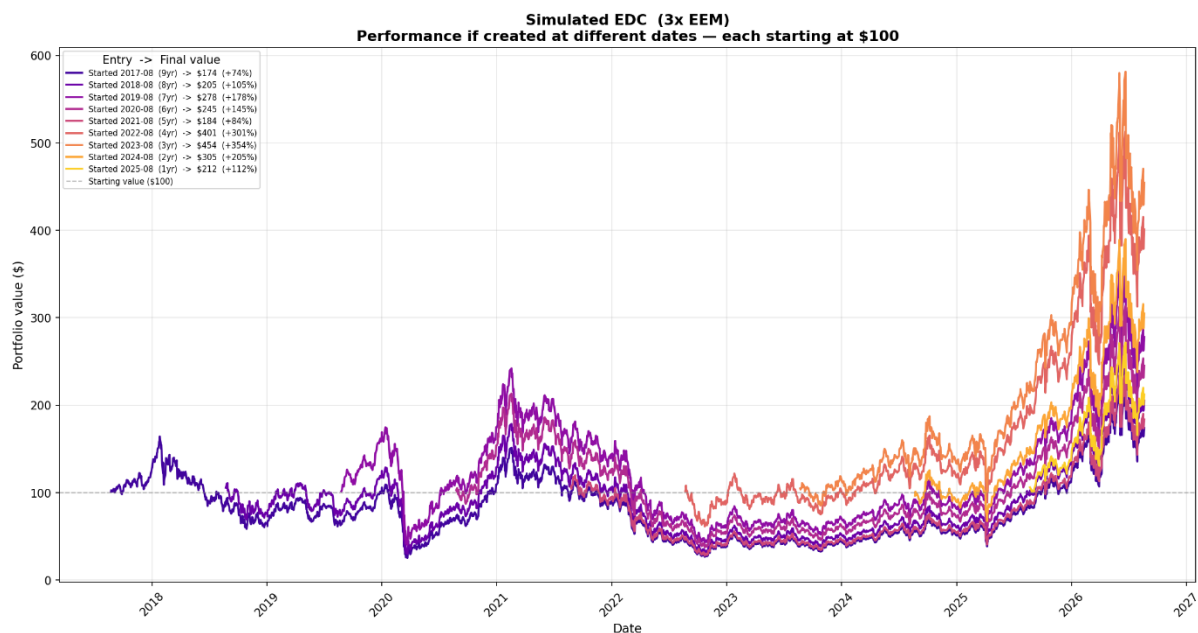






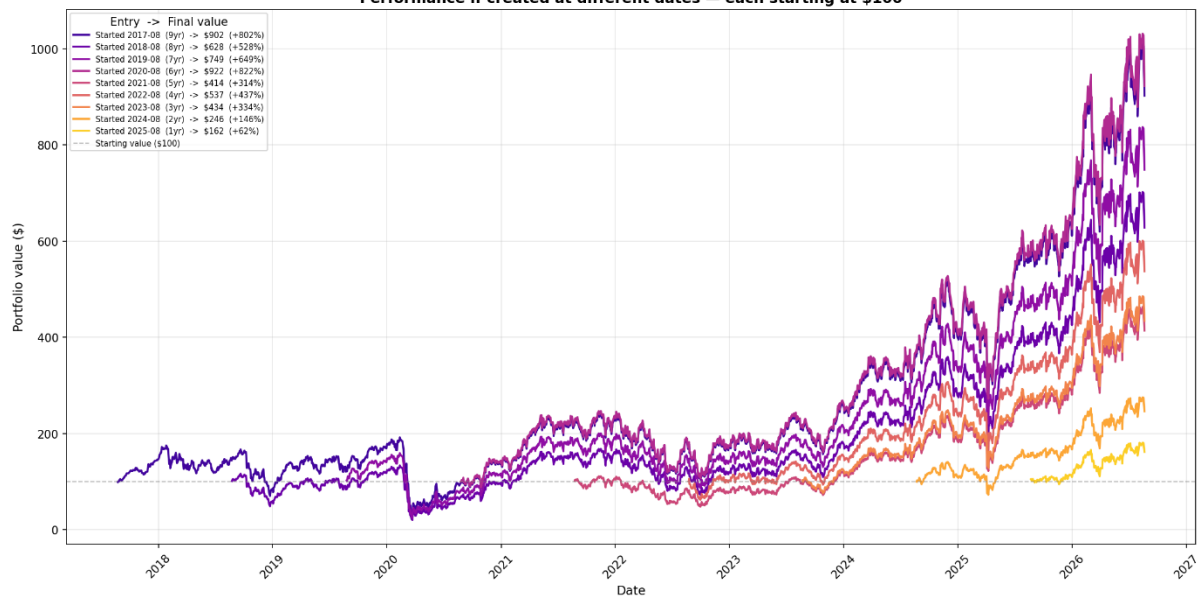




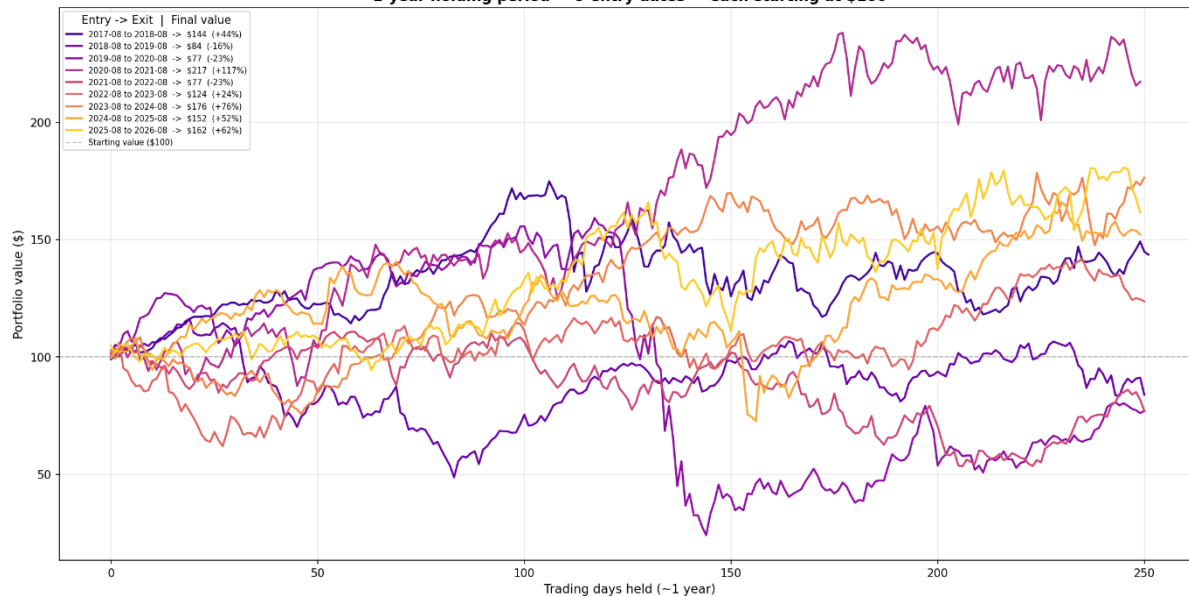


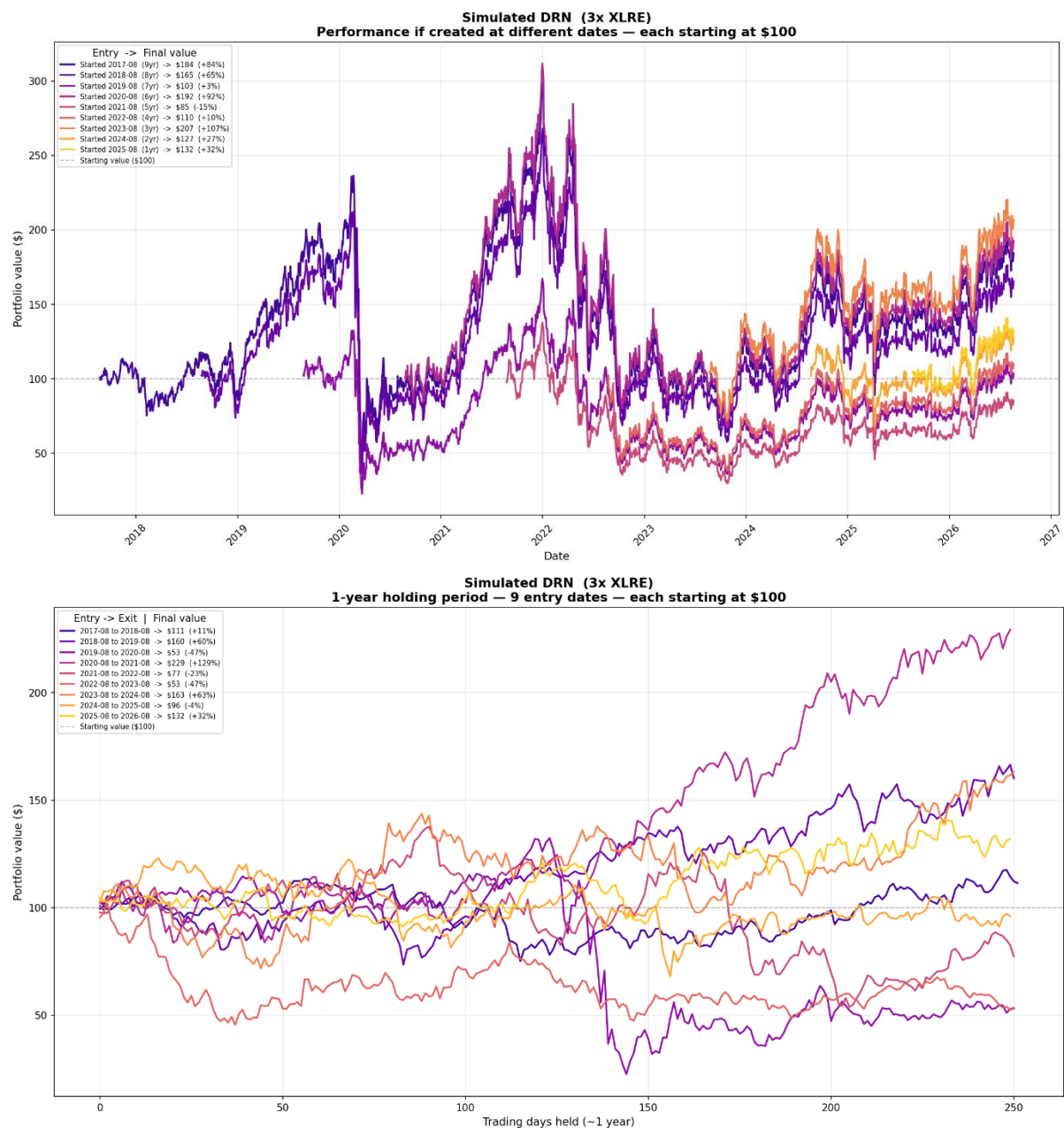


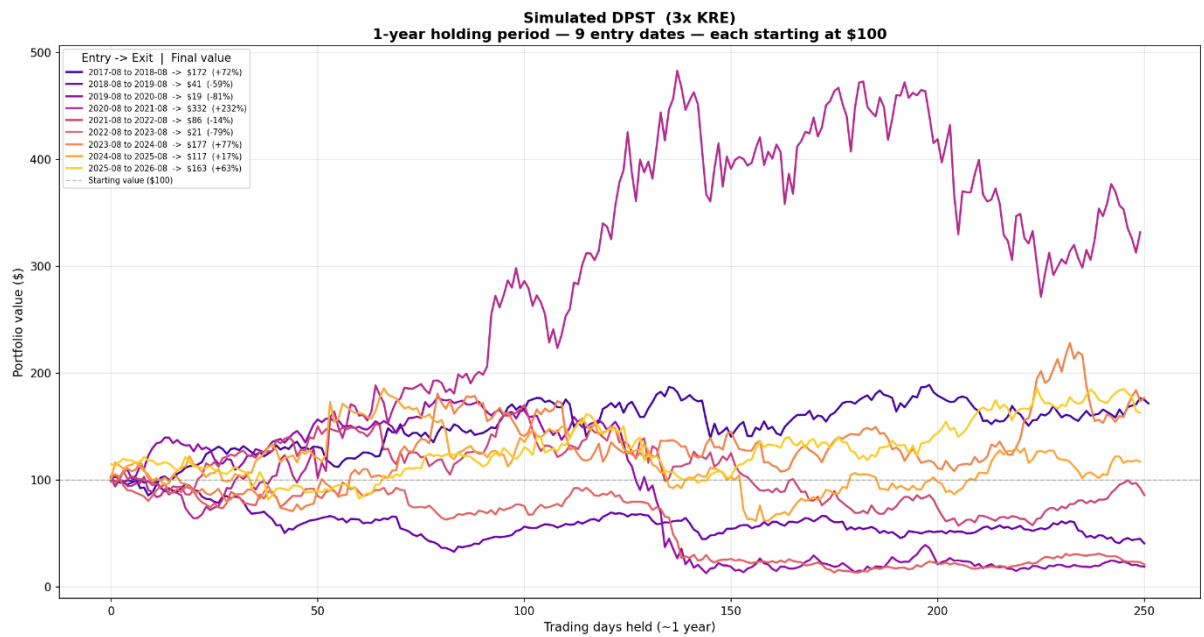
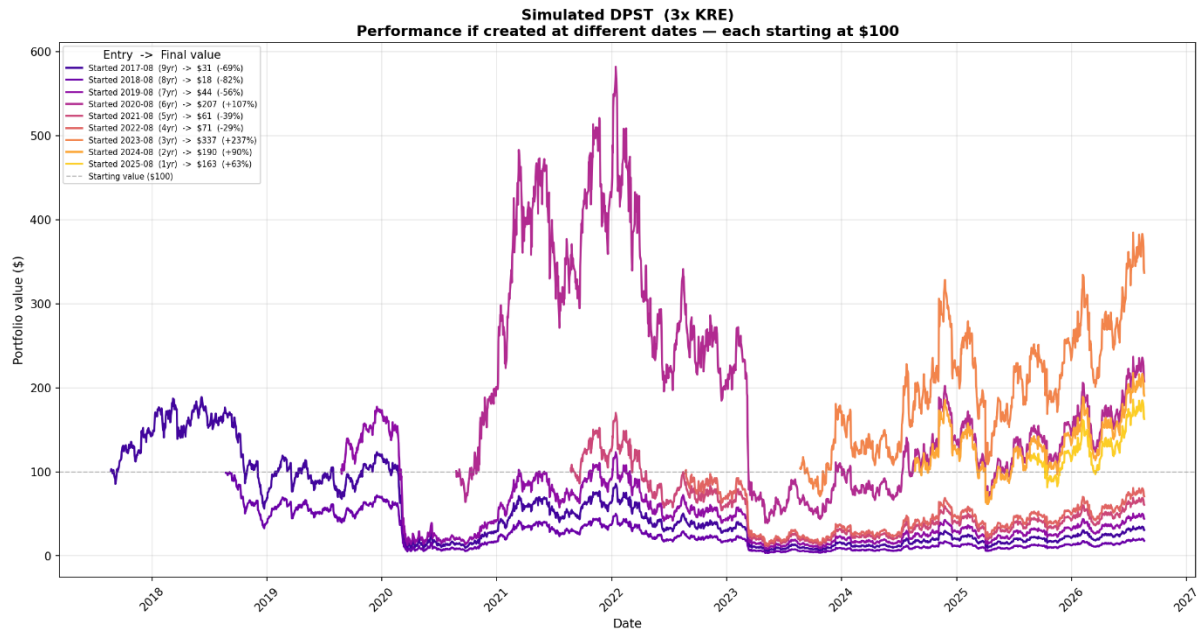
Simulated DUSL (3x XLI)
Performance if created at different dates — each starting at \$100

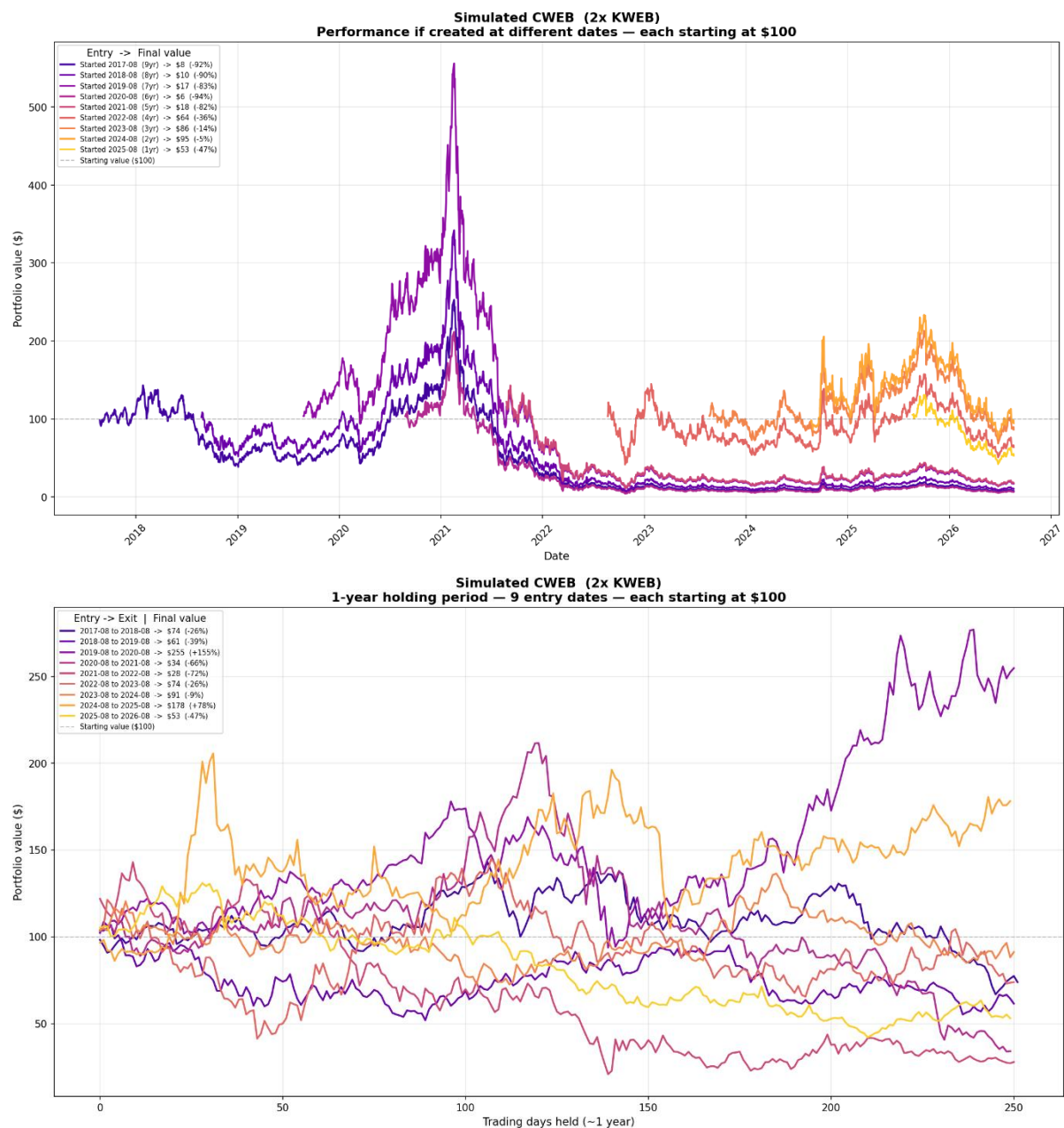


Simulated DUSL (3x XLI)
1-year holding period — 9 entry dates — each starting at \$100

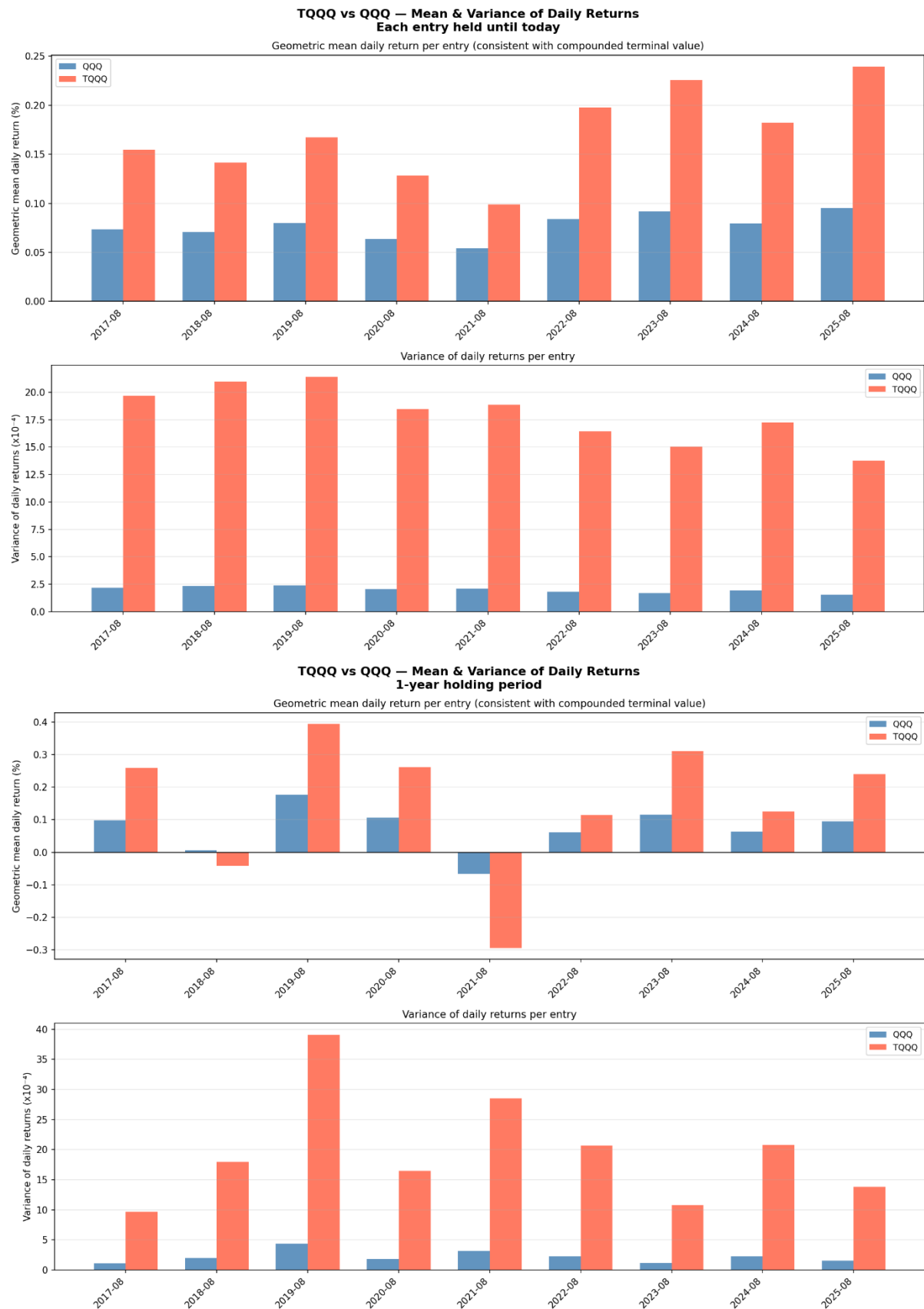




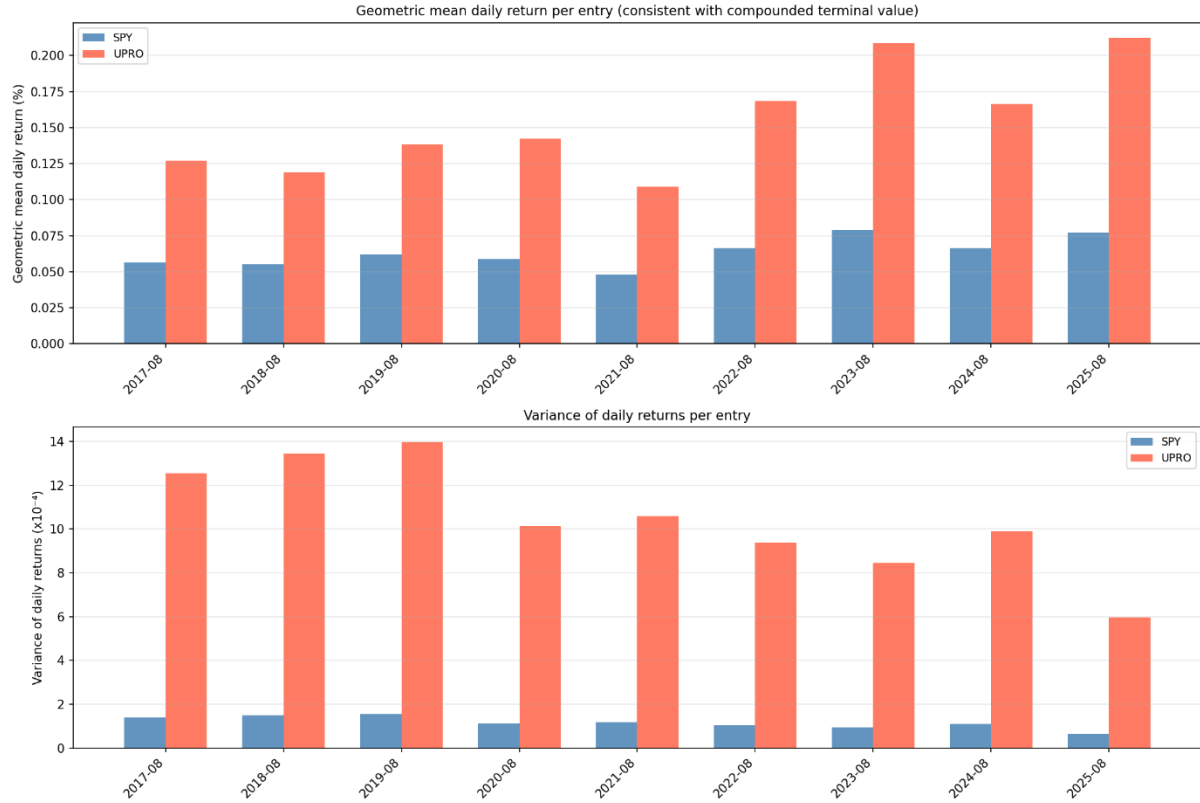




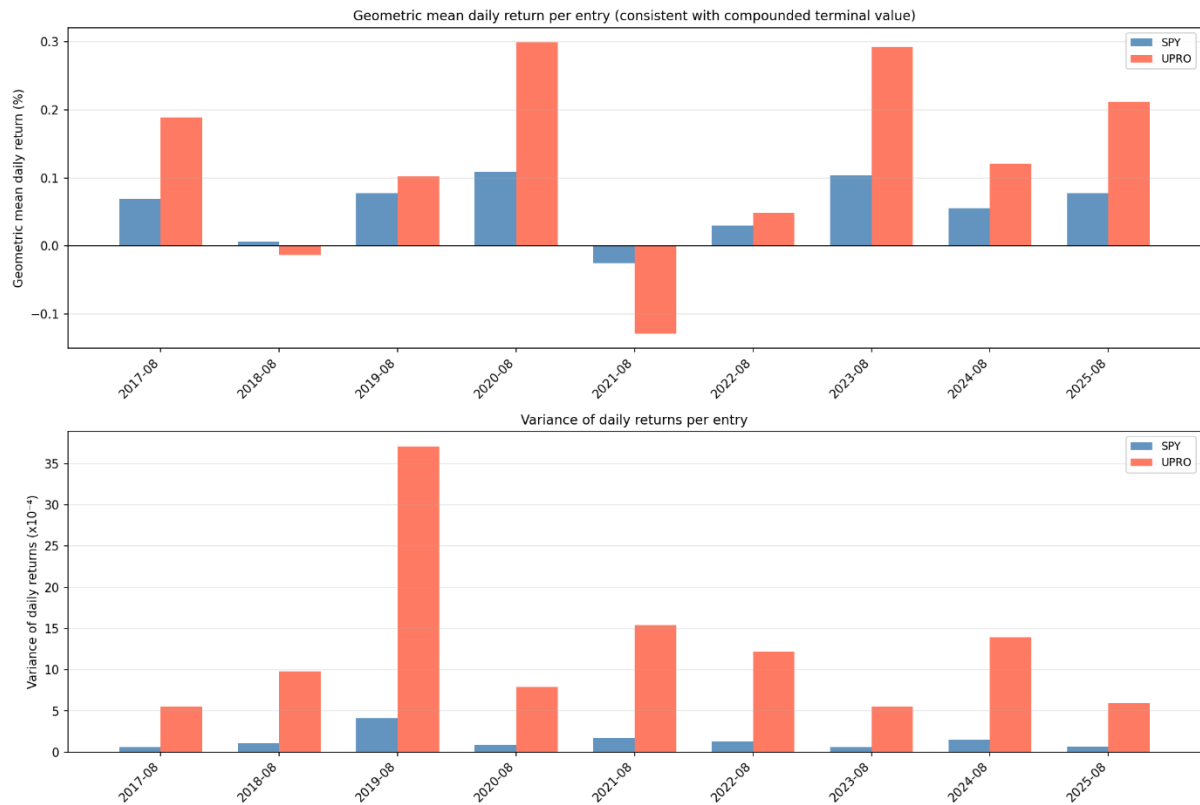
4.2. Statistics



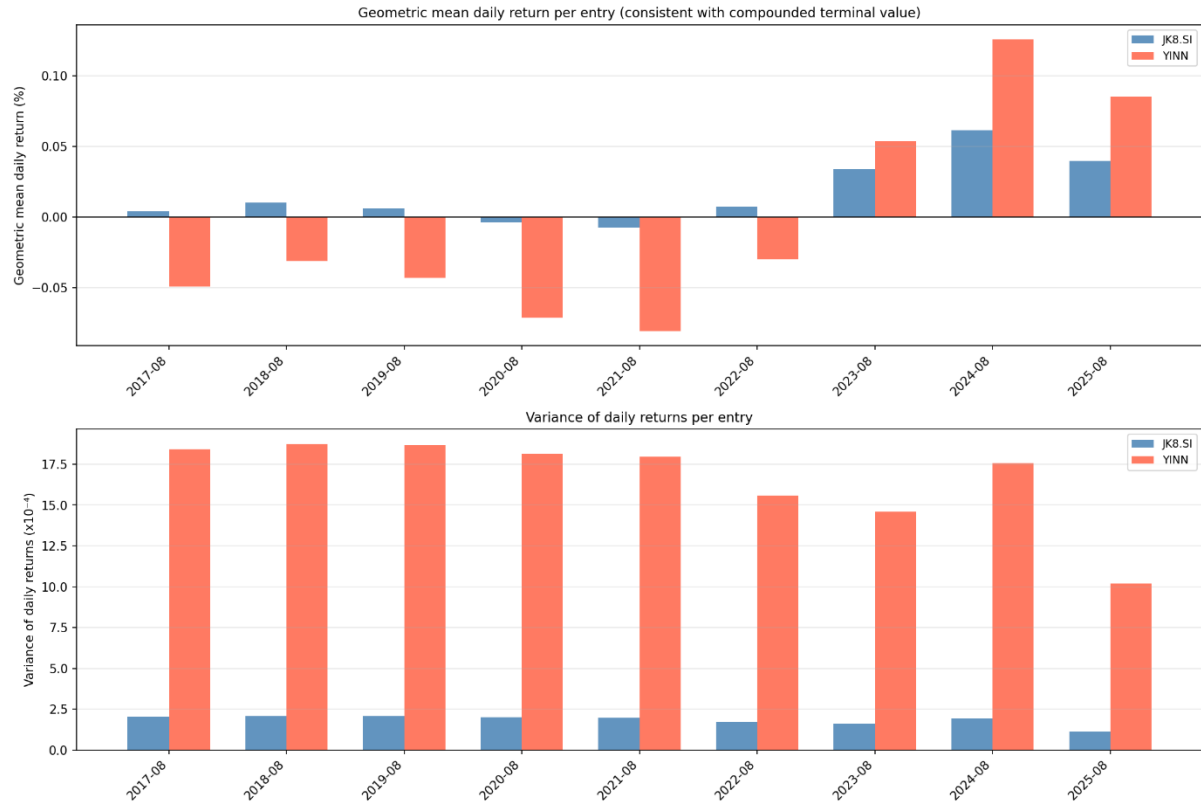
UPRO vs SPY — Mean & Variance of Daily Returns Each entry held until today



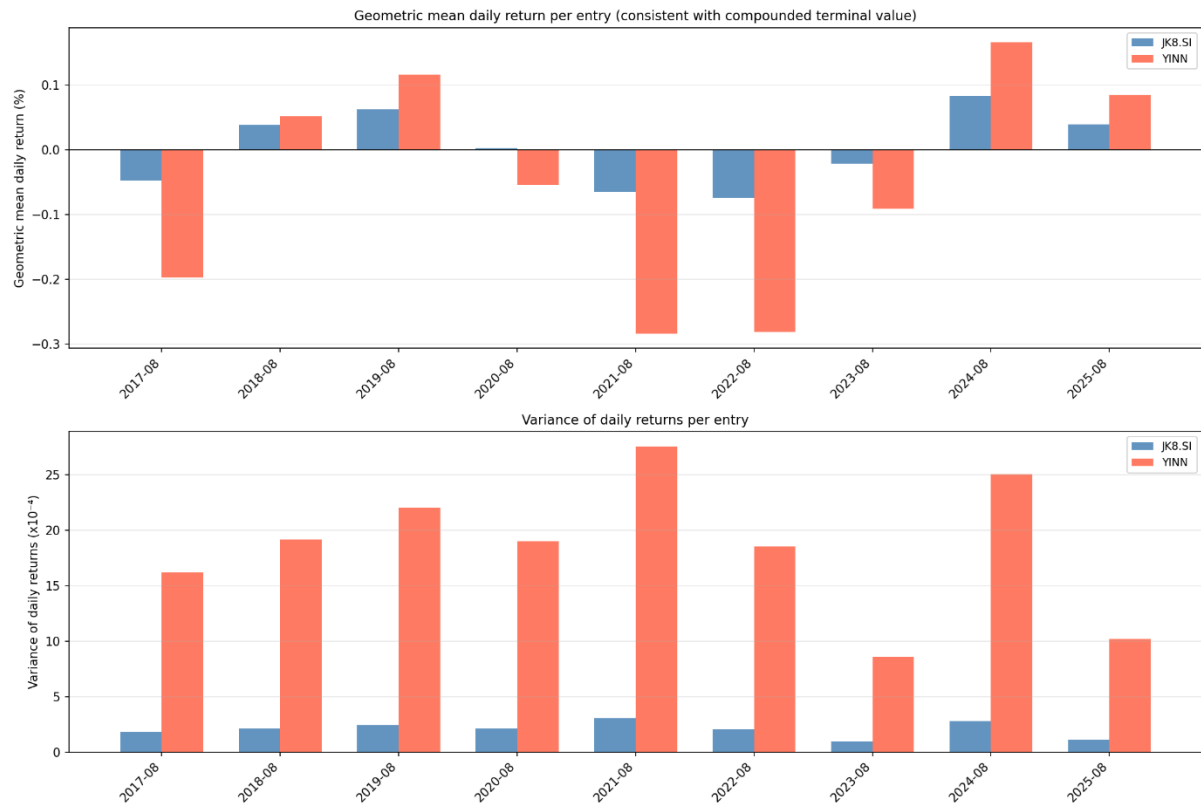
UPRO vs SPY — Mean & Variance of Daily Returns 1-year holding period



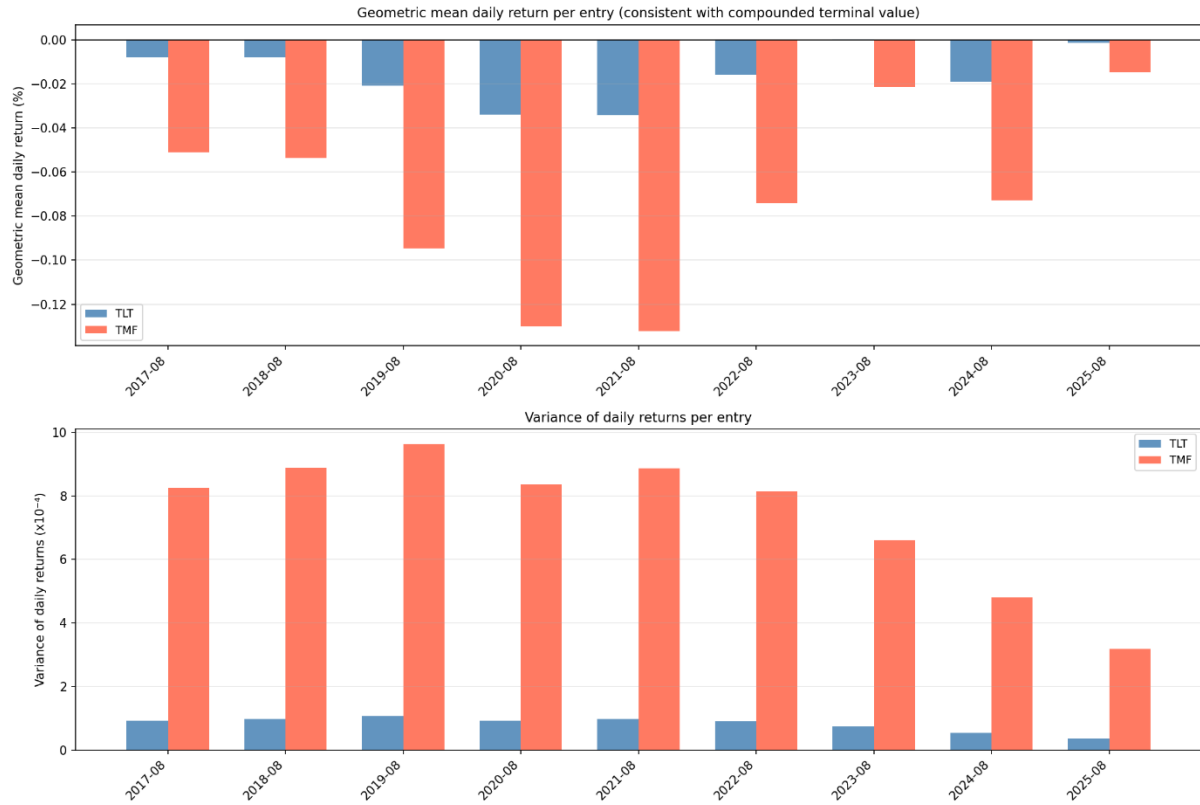
YINN vs JK8.SI — Mean & Variance of Daily Returns Each entry held until today



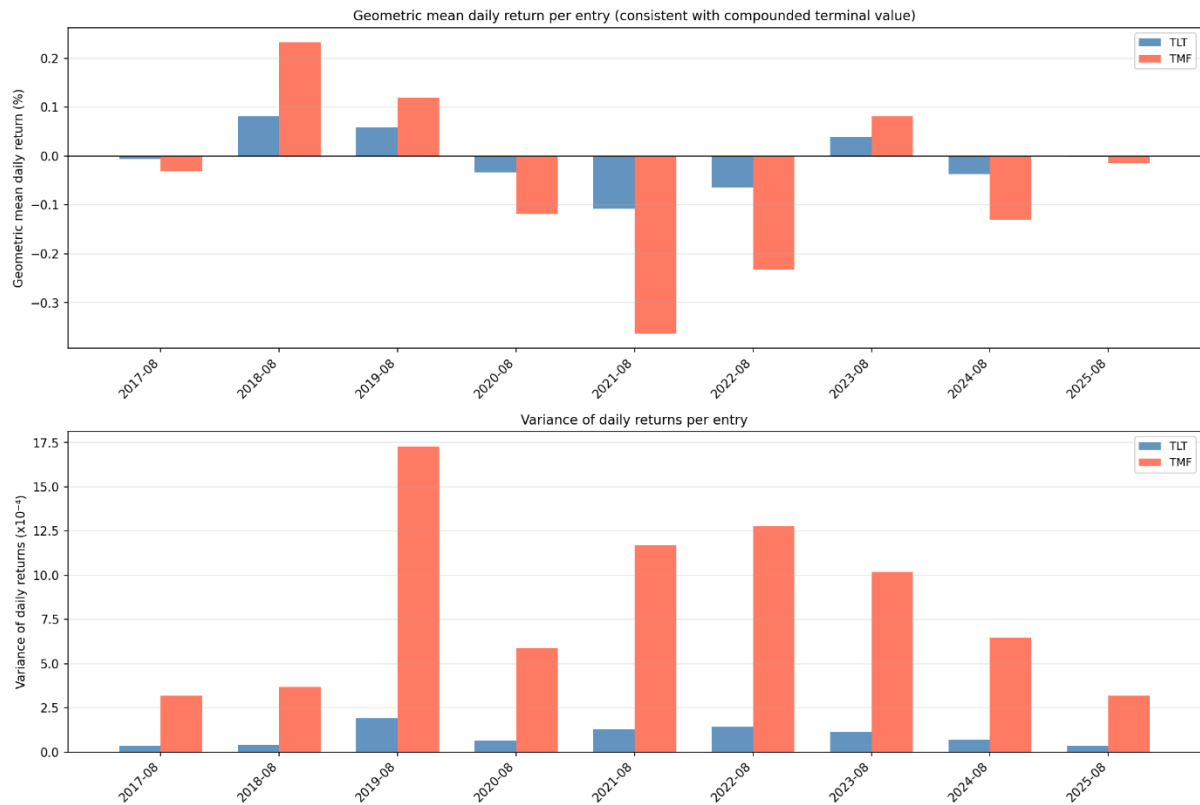
YINN vs JK8.SI — Mean & Variance of Daily Returns 1-year holding period



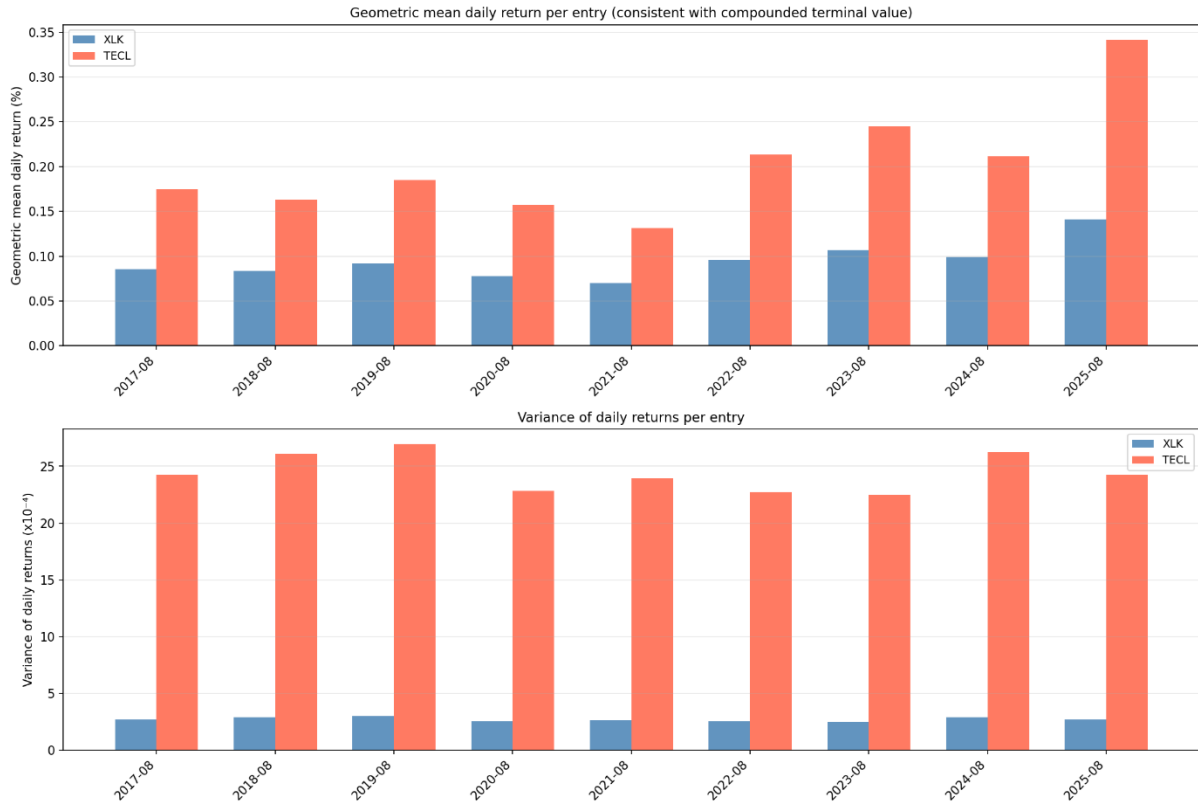
TMF vs TLT — Mean & Variance of Daily Returns Each entry held until today



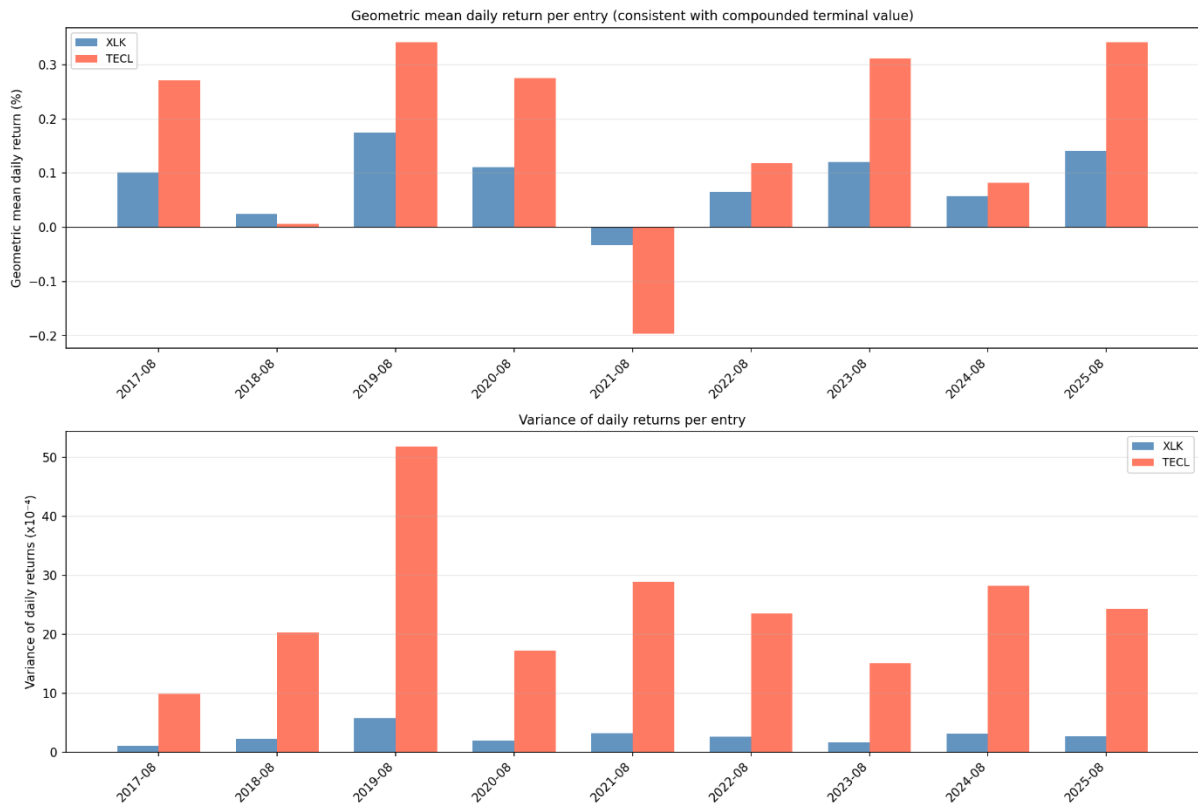
TMF vs TLT — Mean & Variance of Daily Returns 1-year holding period



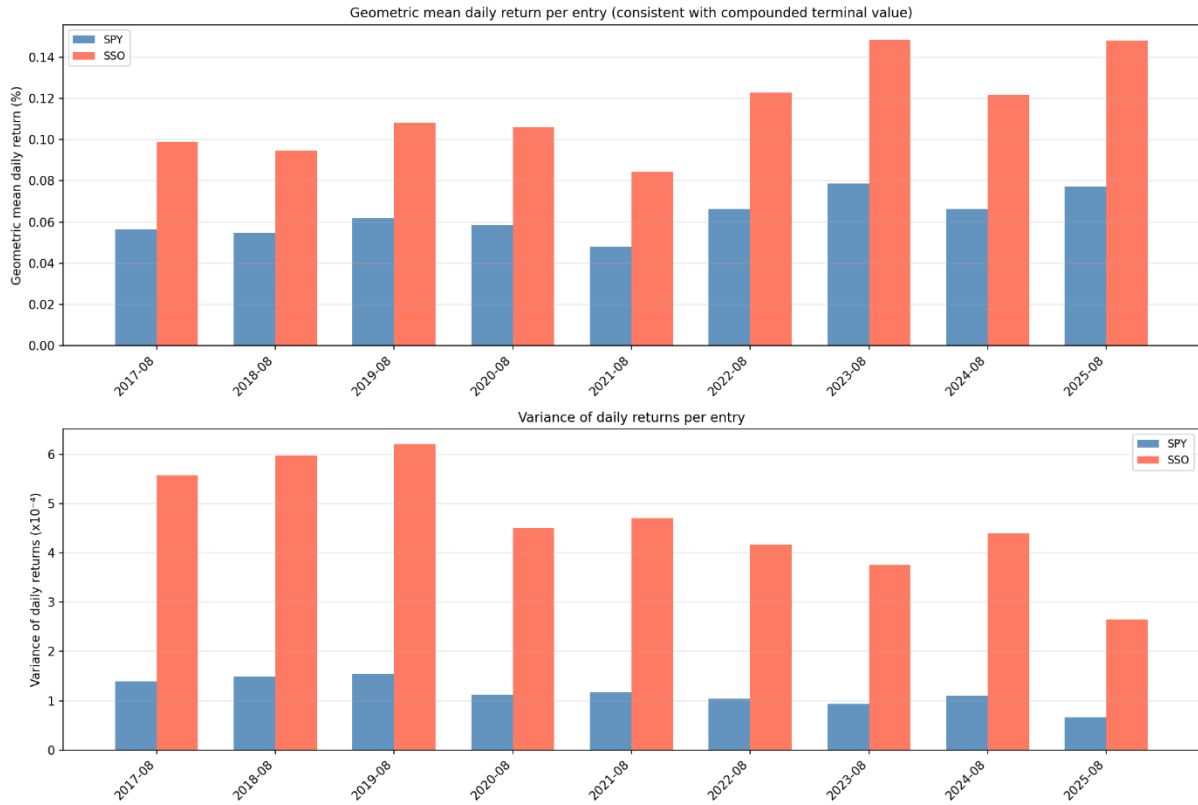
TECL vs XLK — Mean & Variance of Daily Returns Each entry held until today



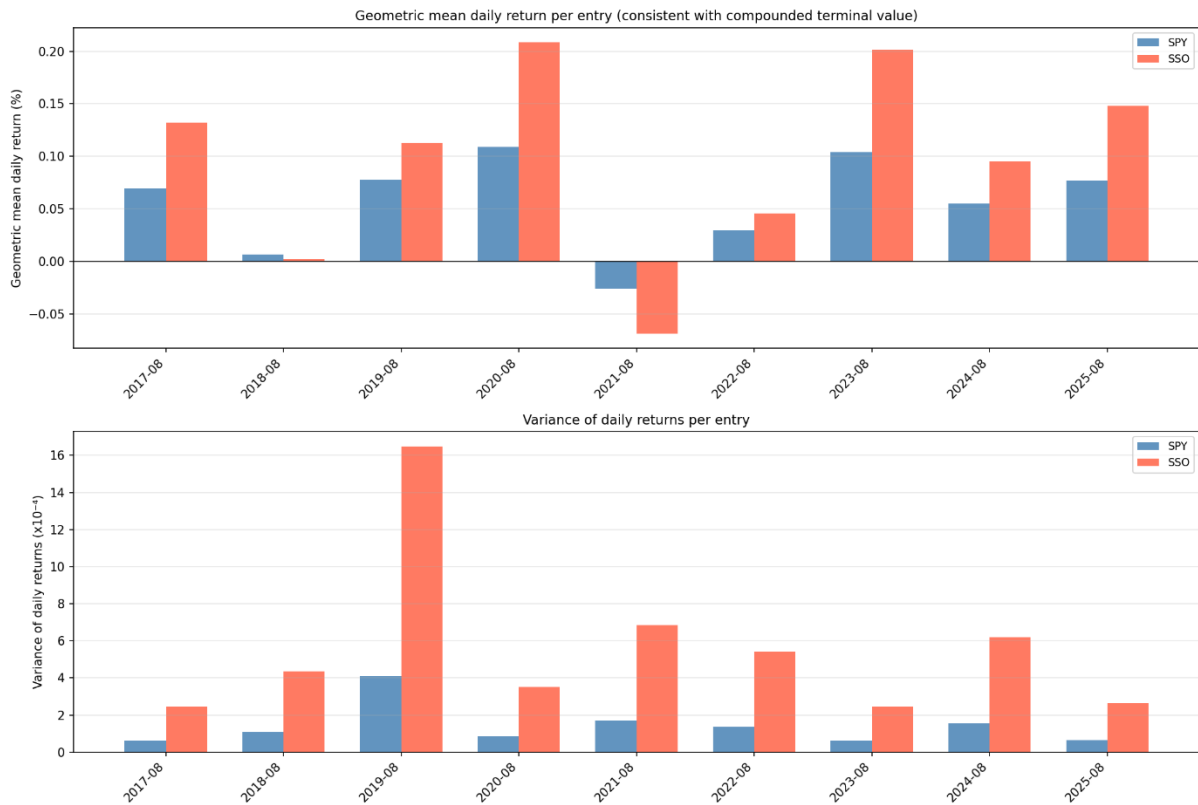
TECL vs XLK — Mean & Variance of Daily Returns 1-year holding period



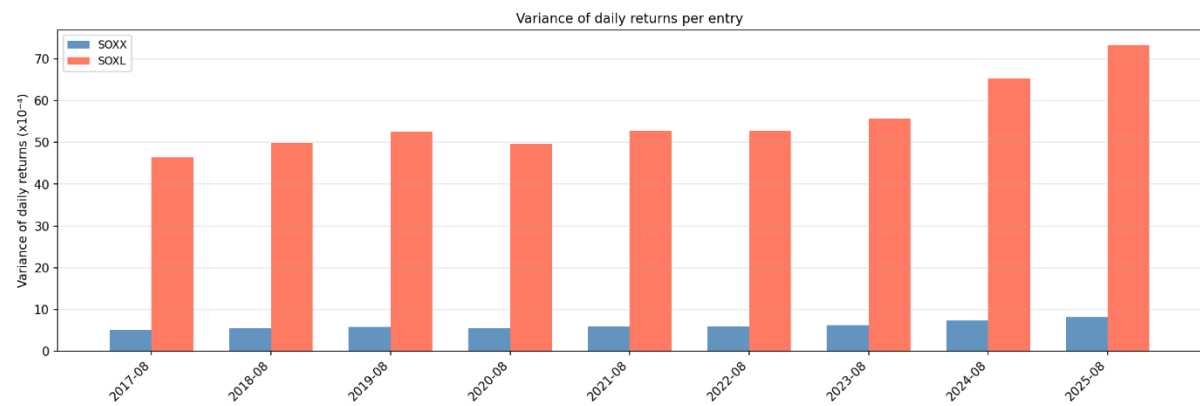
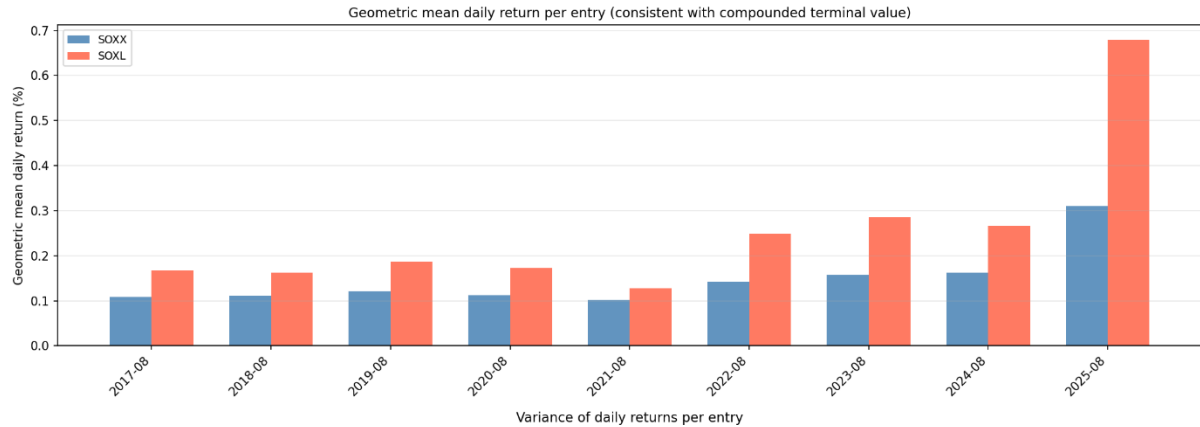
SSO vs SPY – Mean & Variance of Daily Returns Each entry held until today



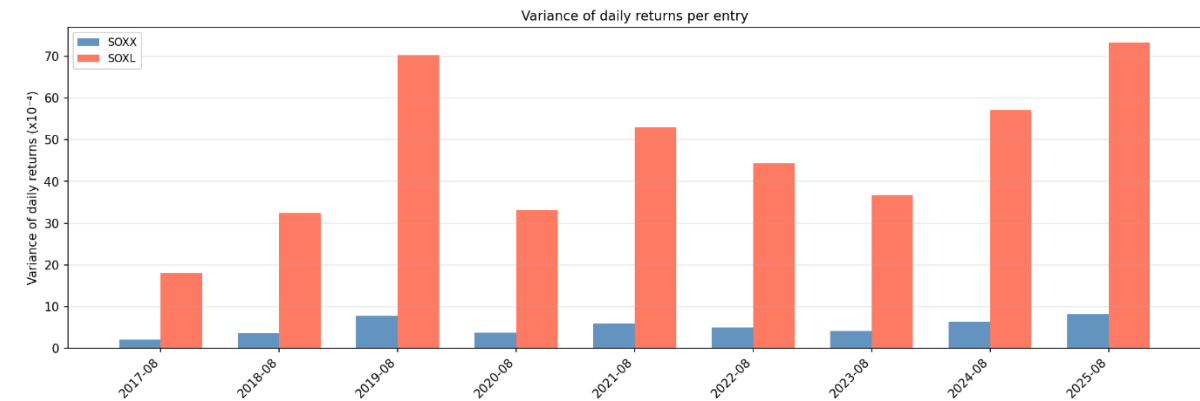
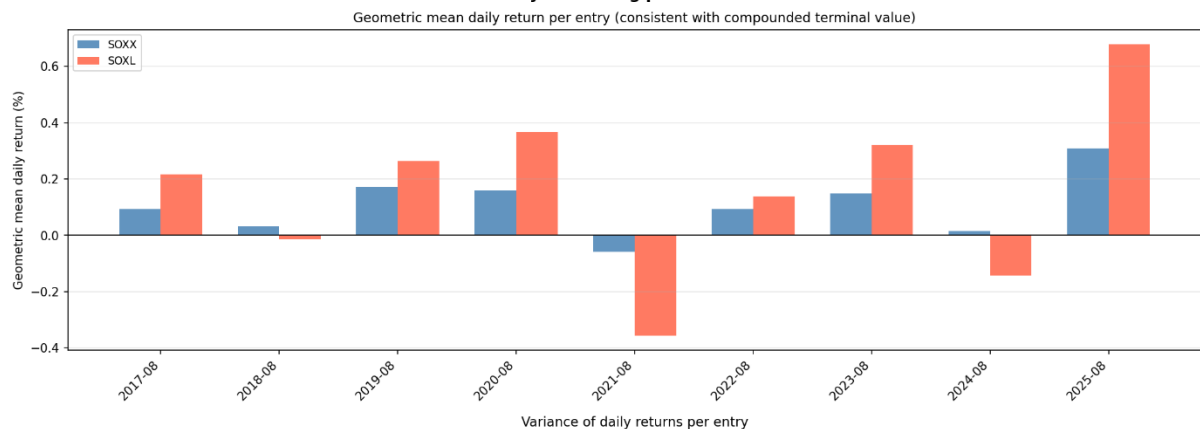
SSO vs SPY – Mean & Variance of Daily Returns 1-year holding period



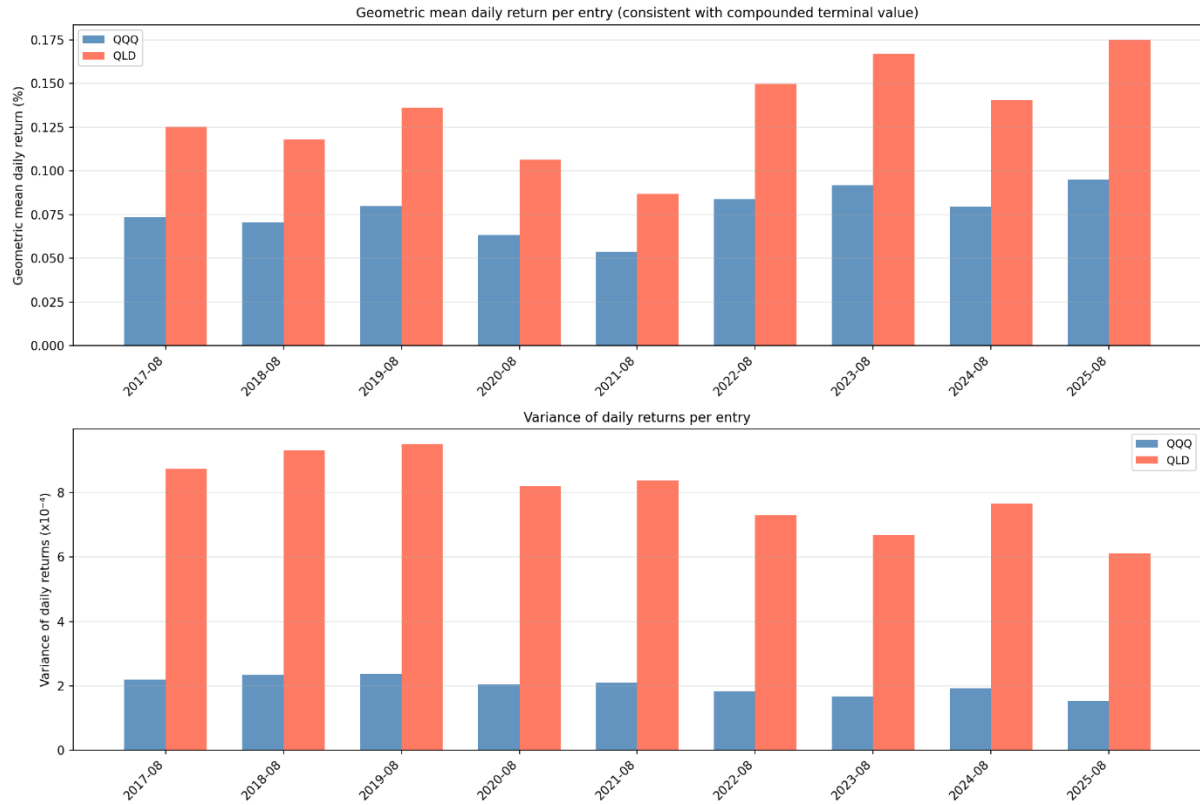
SOXL vs SOXX — Mean & Variance of Daily Returns Each entry held until today



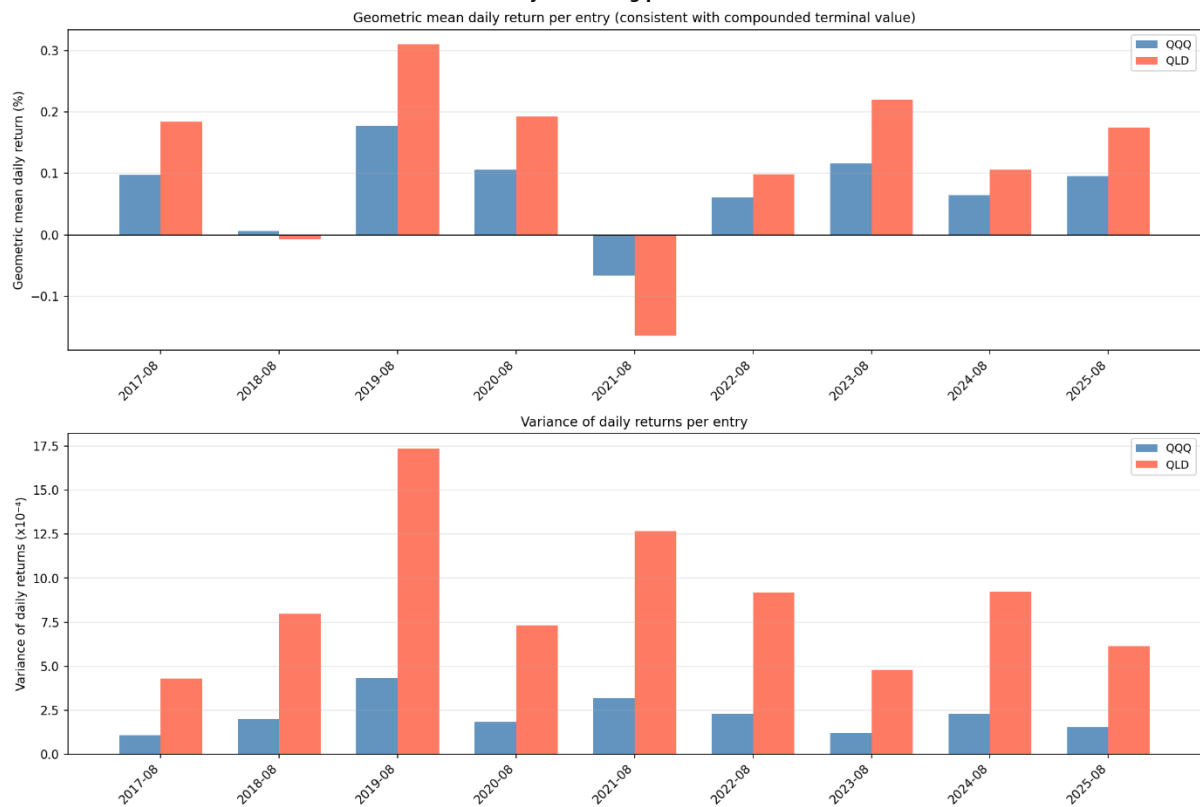
SOXL vs SOXX — Mean & Variance of Daily Returns 1-year holding period



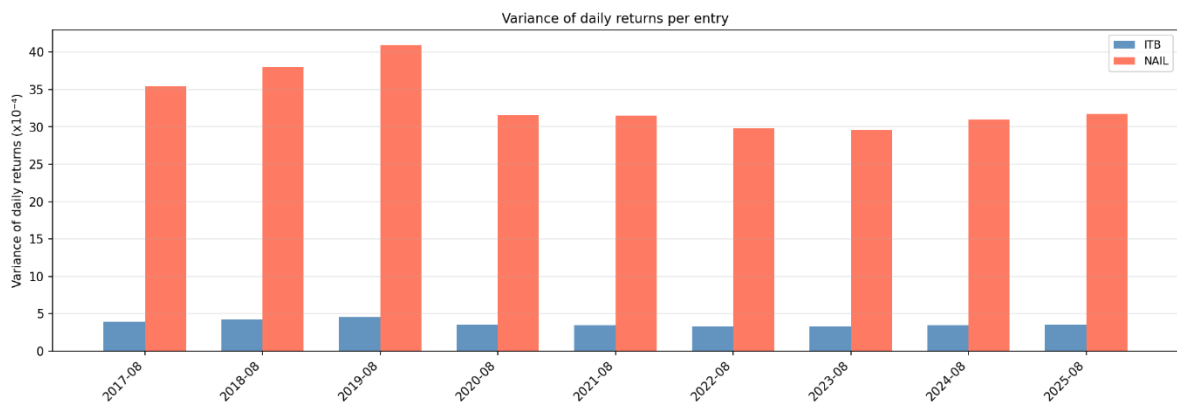
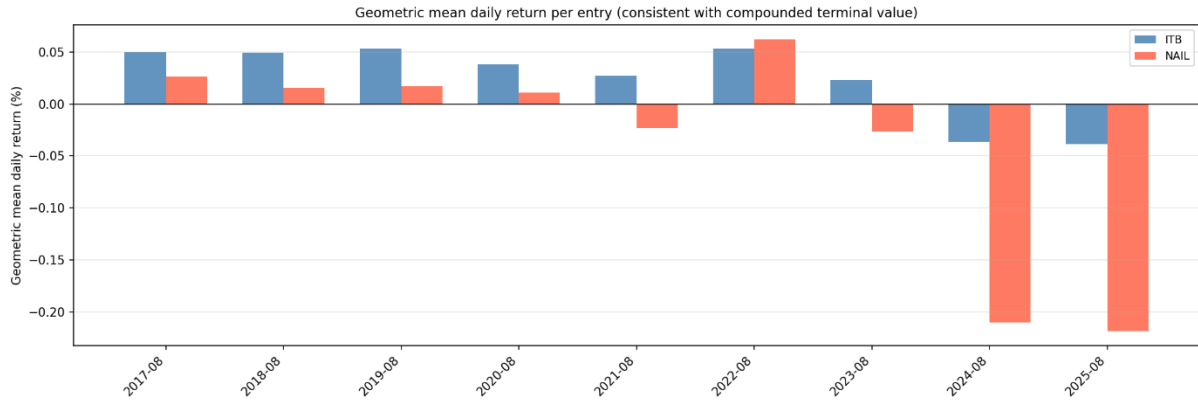
QLD vs QQQ — Mean & Variance of Daily Returns Each entry held until today



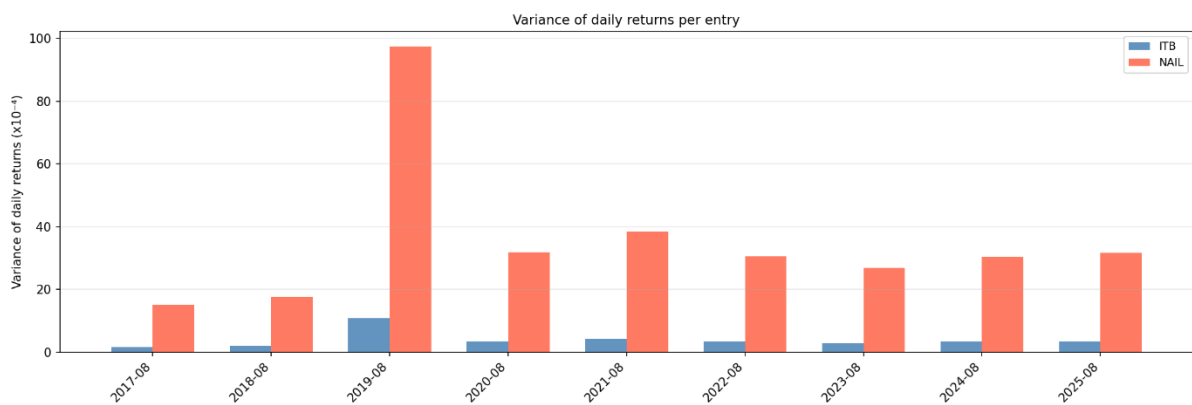
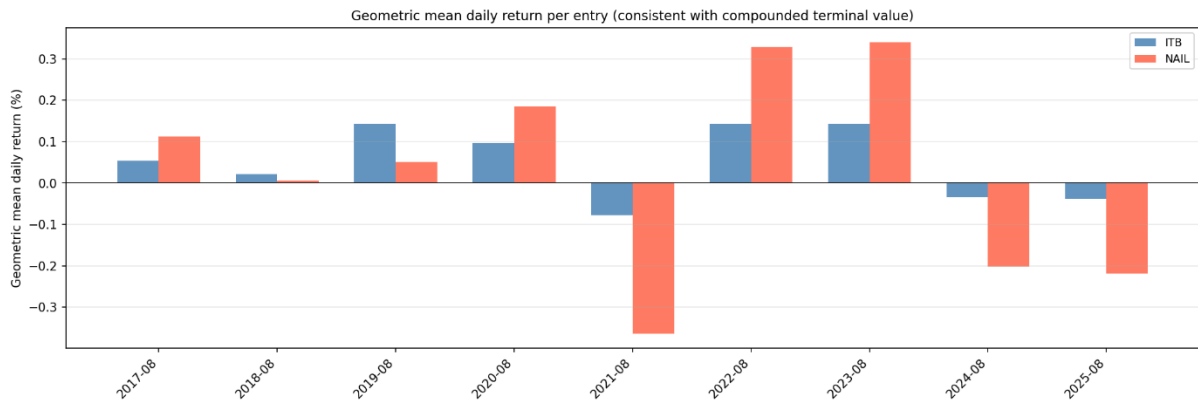
QLD vs QQQ — Mean & Variance of Daily Returns 1-year holding period



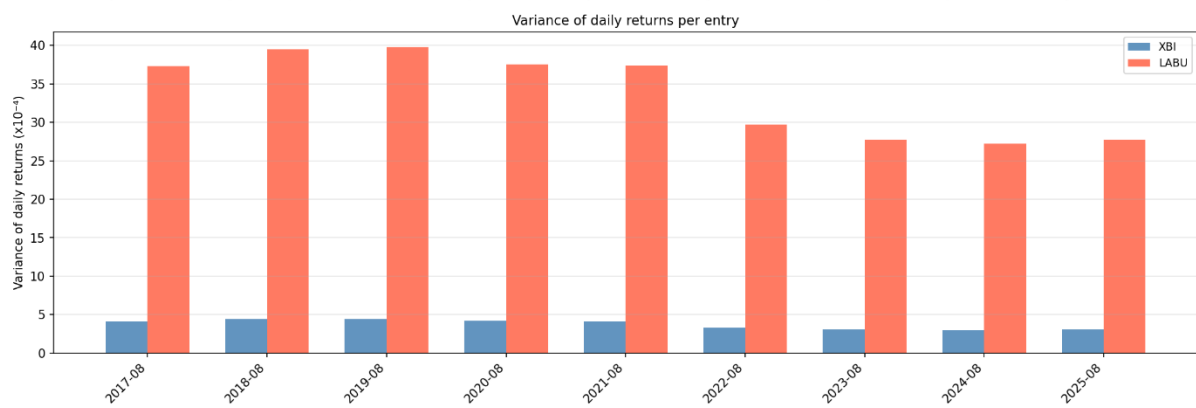
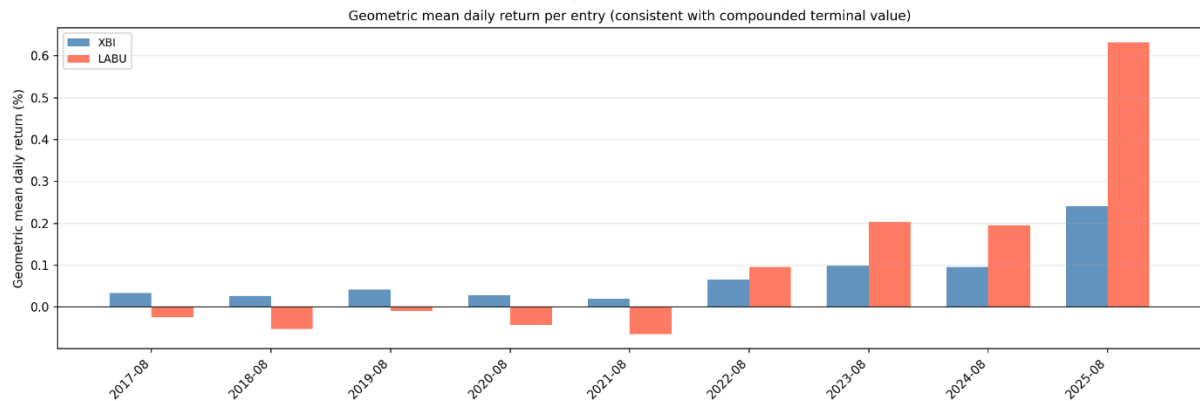
NAIL vs ITB — Mean & Variance of Daily Returns Each entry held until today



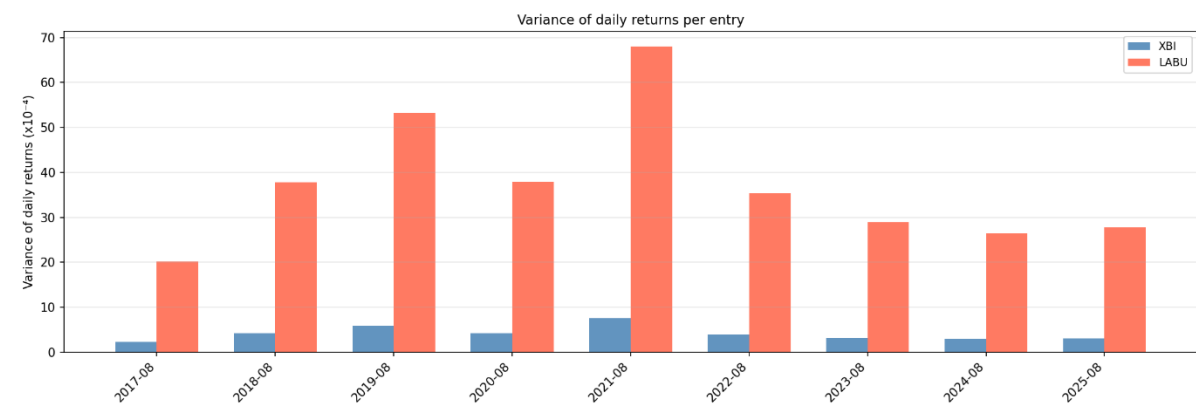
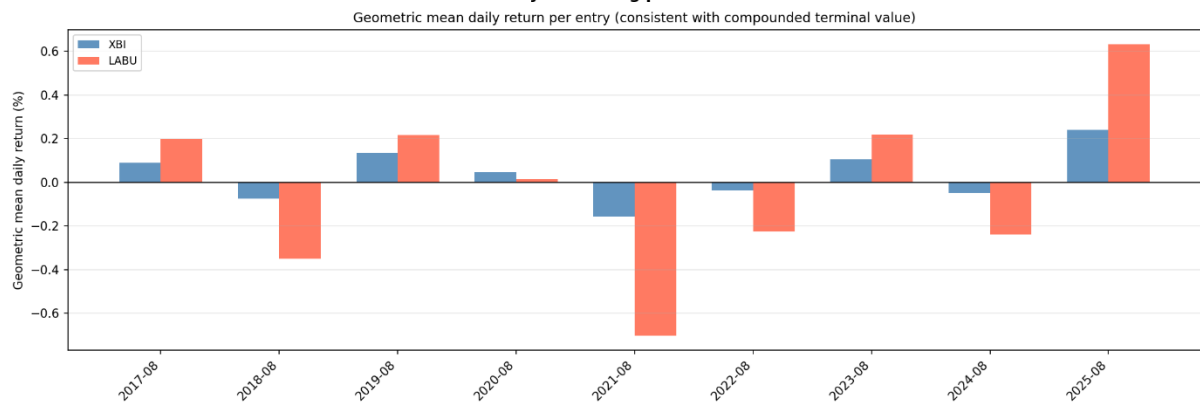
NAIL vs ITB — Mean & Variance of Daily Returns 1-year holding period



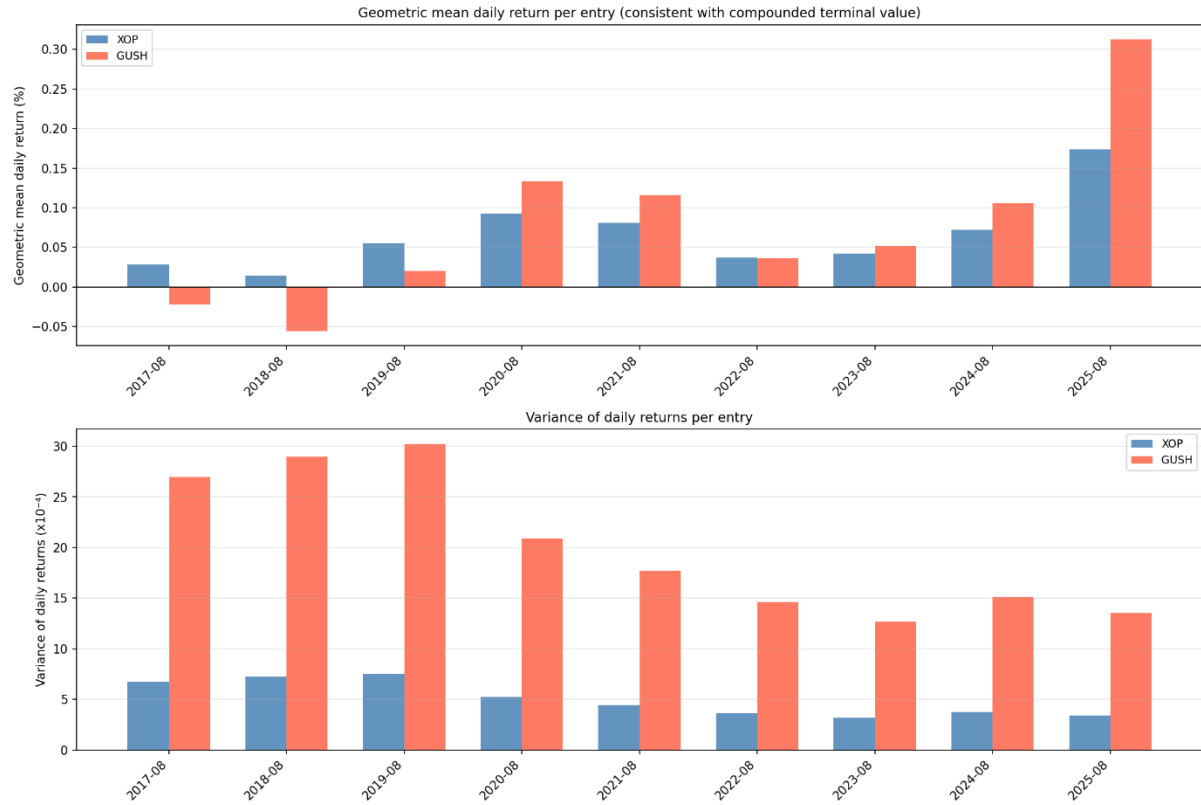
LABU vs XBI — Mean & Variance of Daily Returns Each entry held until today



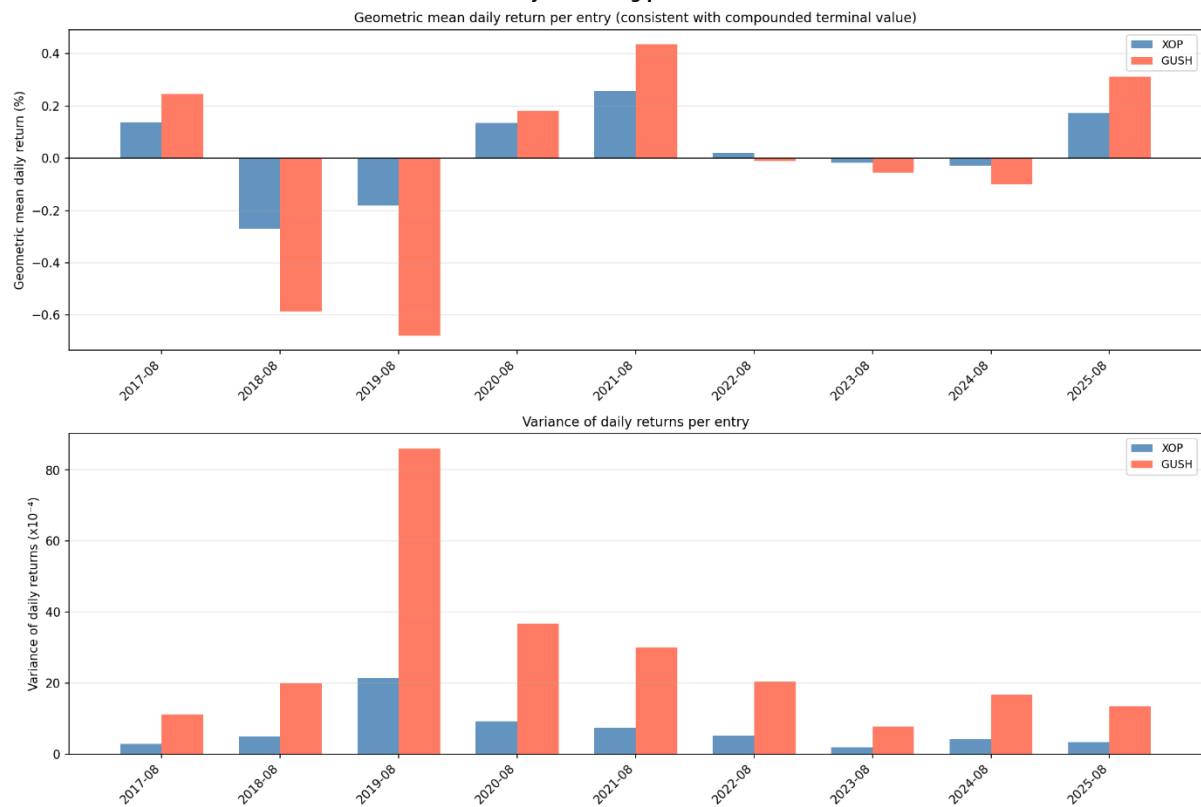
LABU vs XBI — Mean & Variance of Daily Returns 1-year holding period



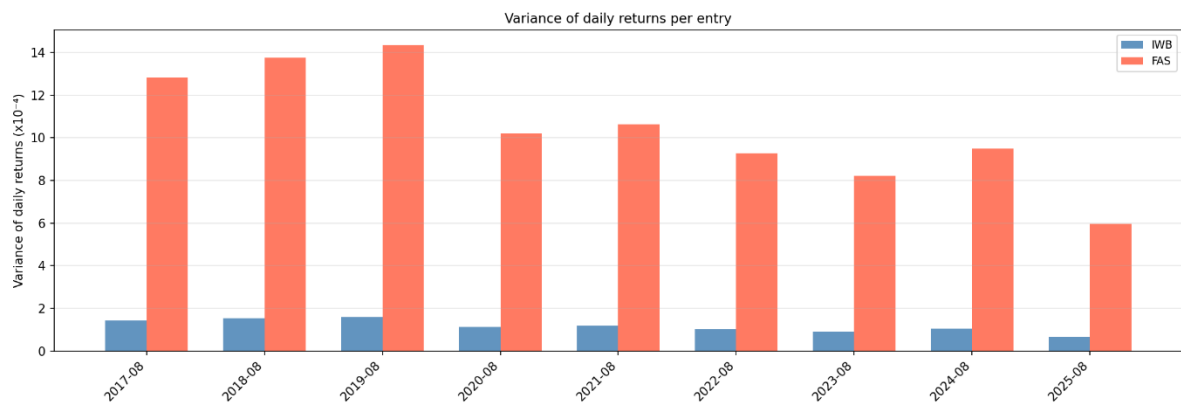
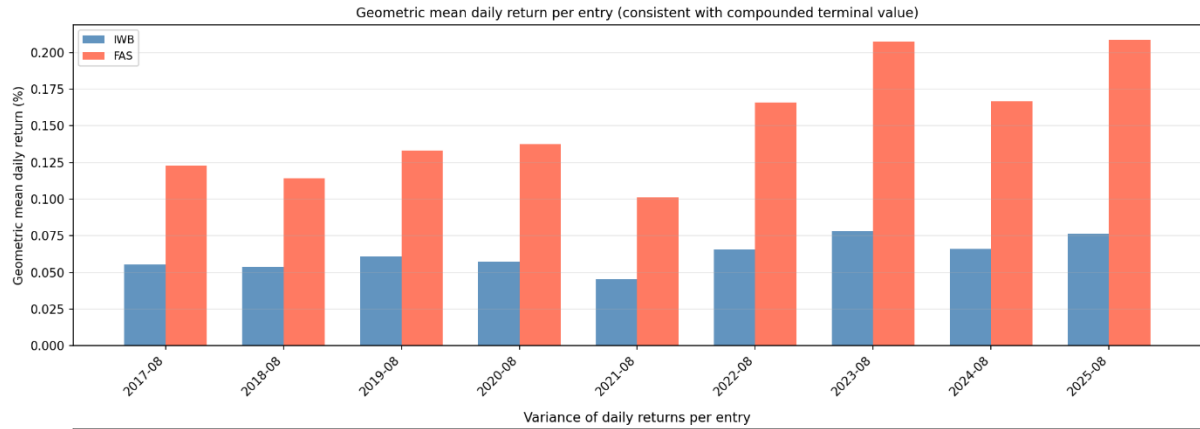
GUSH vs XOP — Mean & Variance of Daily Returns Each entry held until today



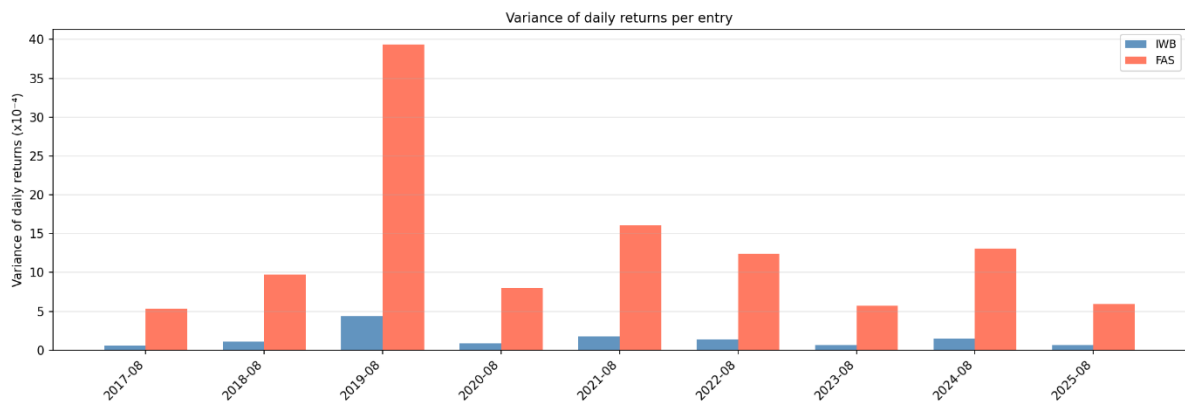
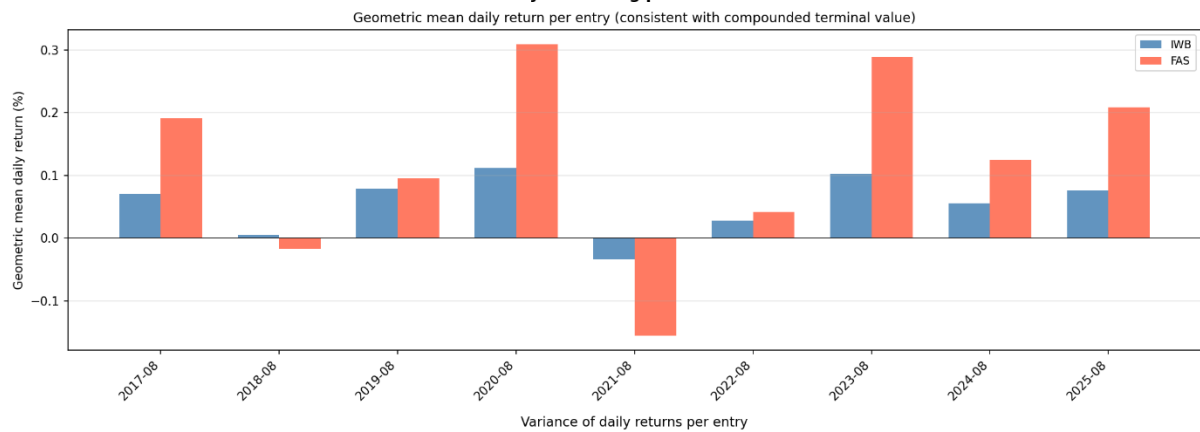
GUSH vs XOP — Mean & Variance of Daily Returns 1-year holding period



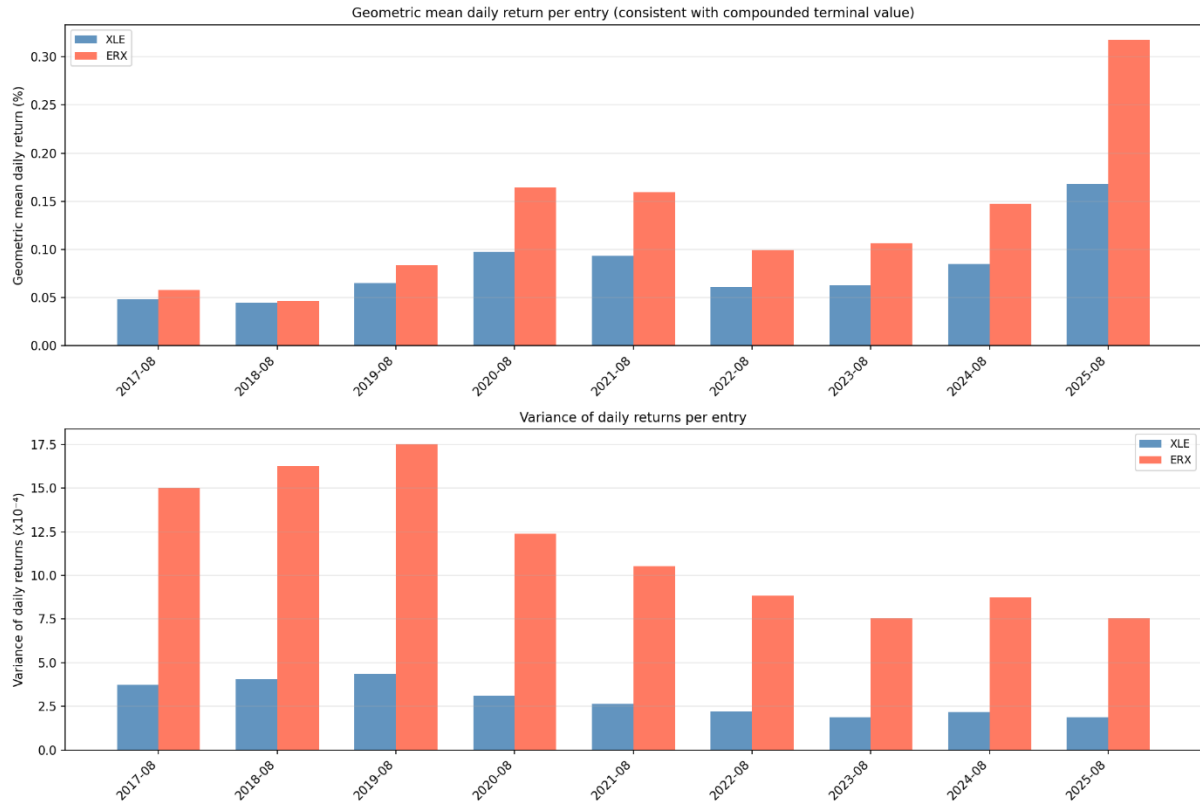
FAS vs IWB – Mean & Variance of Daily Returns Each entry held until today



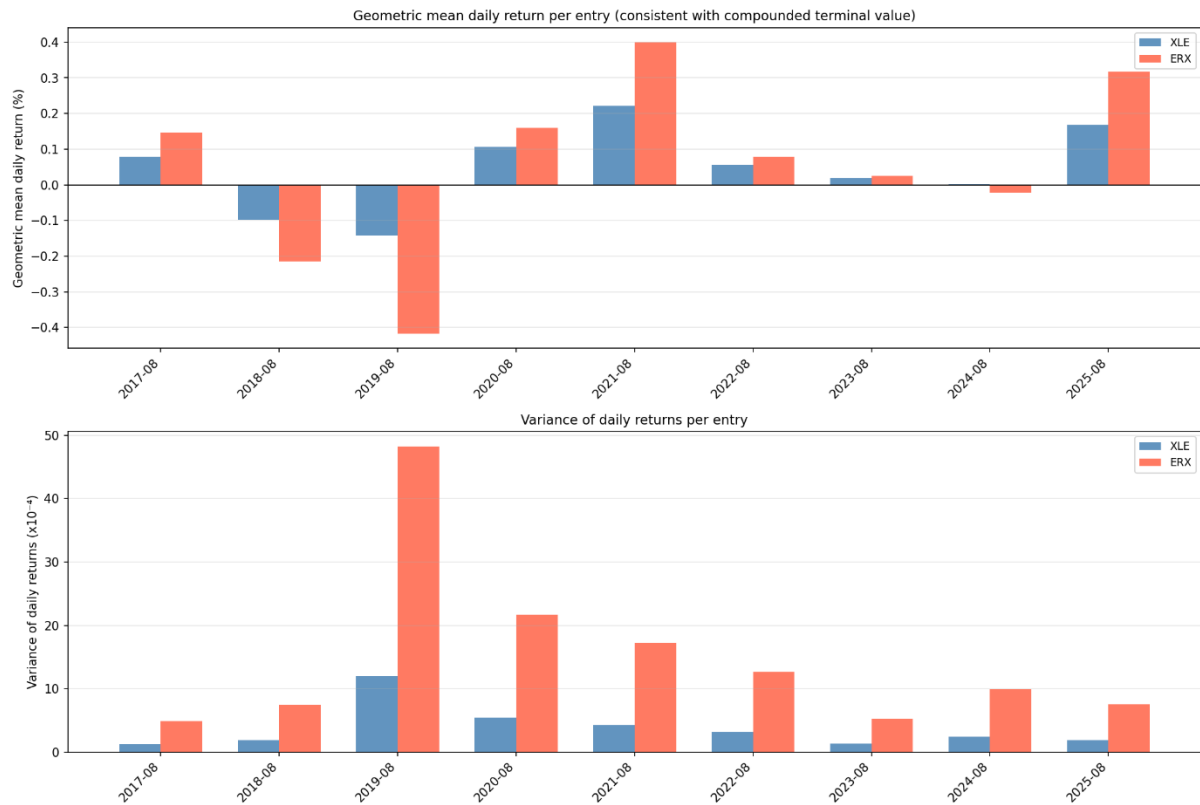
FAS vs IWB – Mean & Variance of Daily Returns 1-year holding period

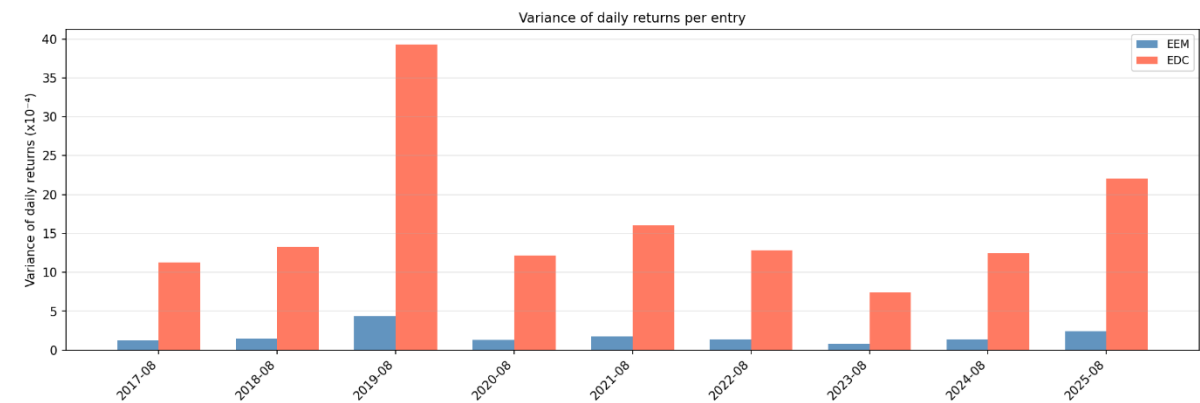
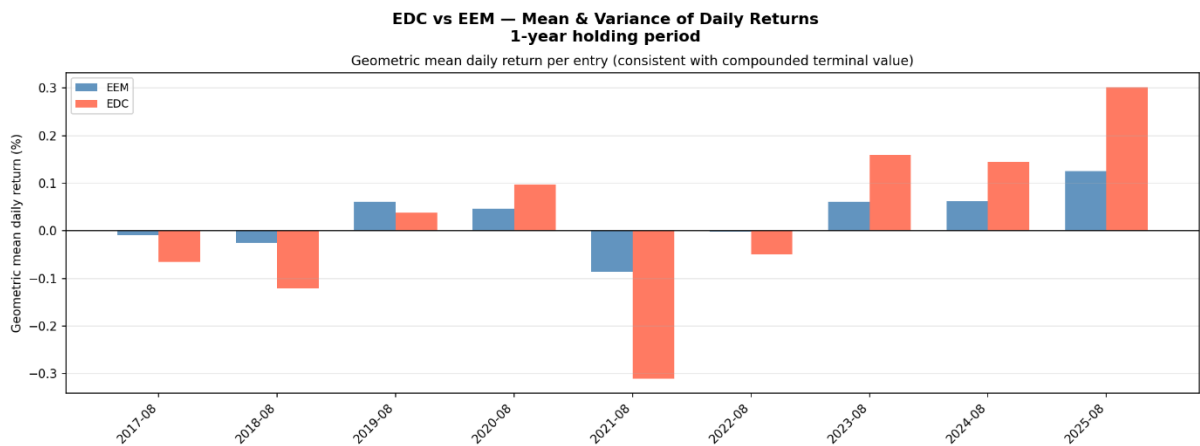
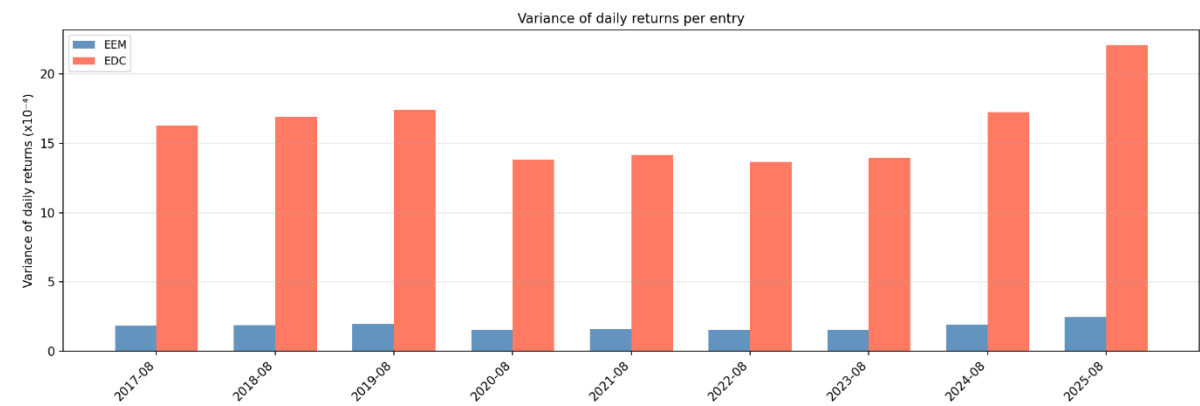
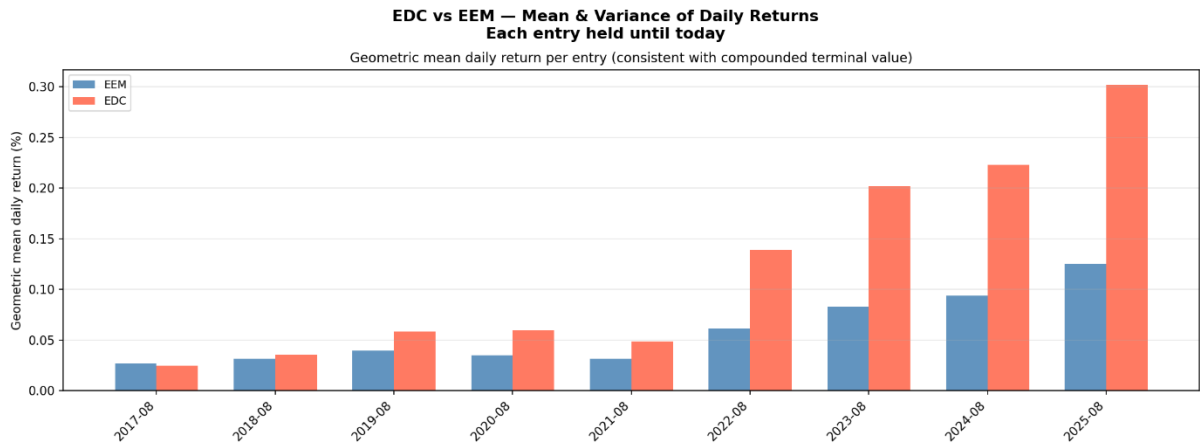


ERX vs XLE — Mean & Variance of Daily Returns Each entry held until today

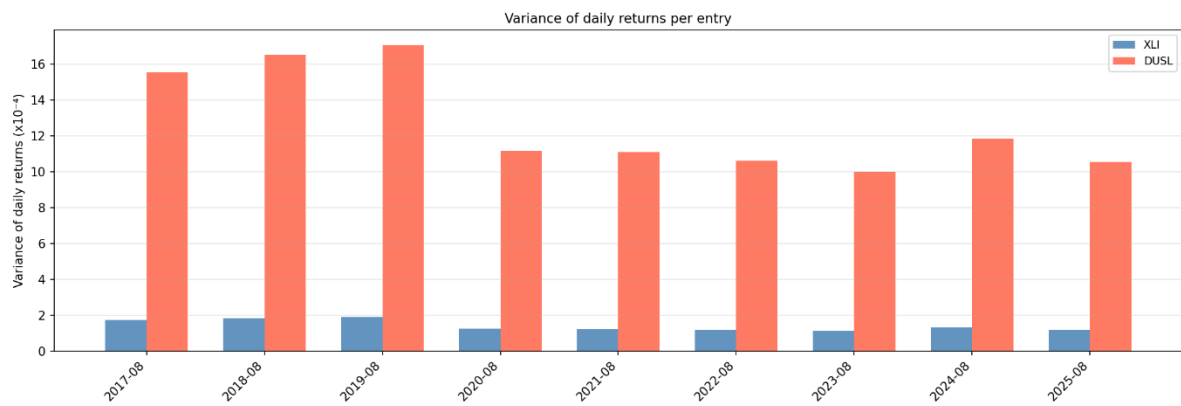
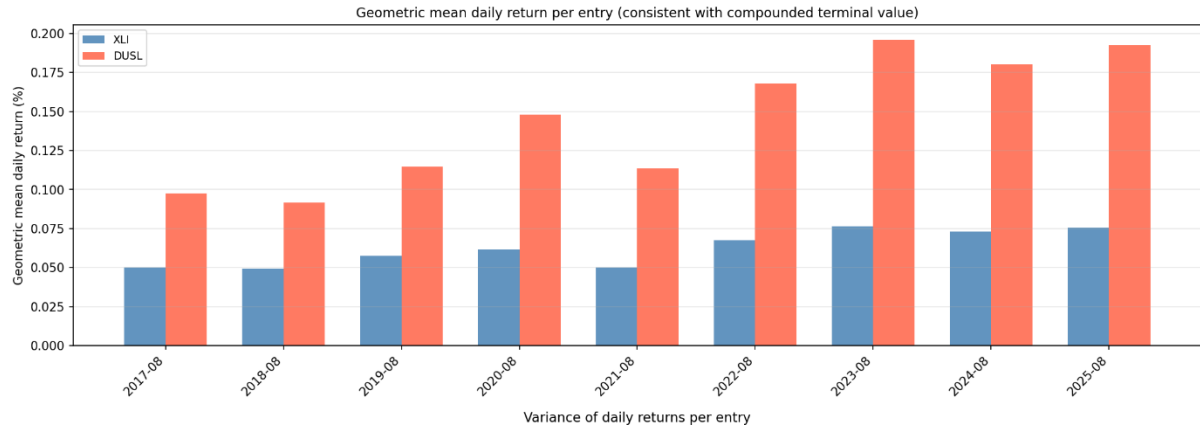


ERX vs XLE — Mean & Variance of Daily Returns 1-year holding period

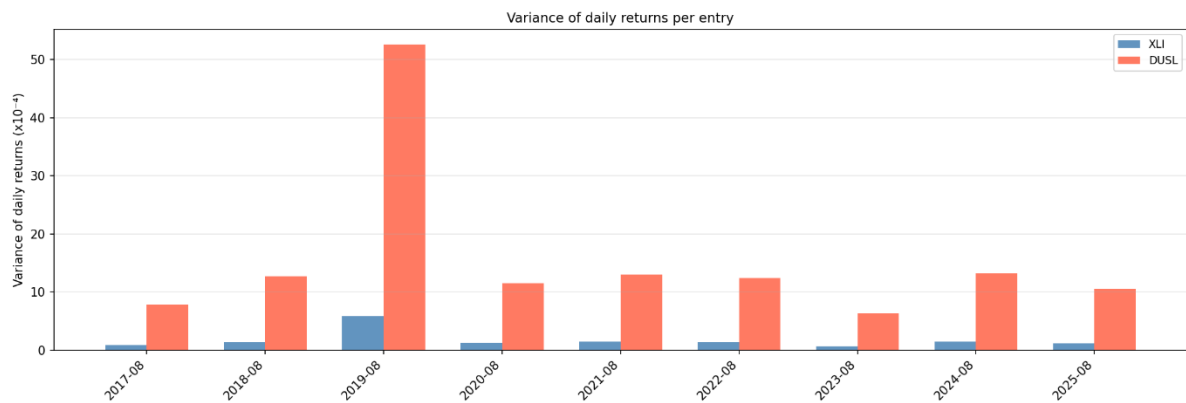
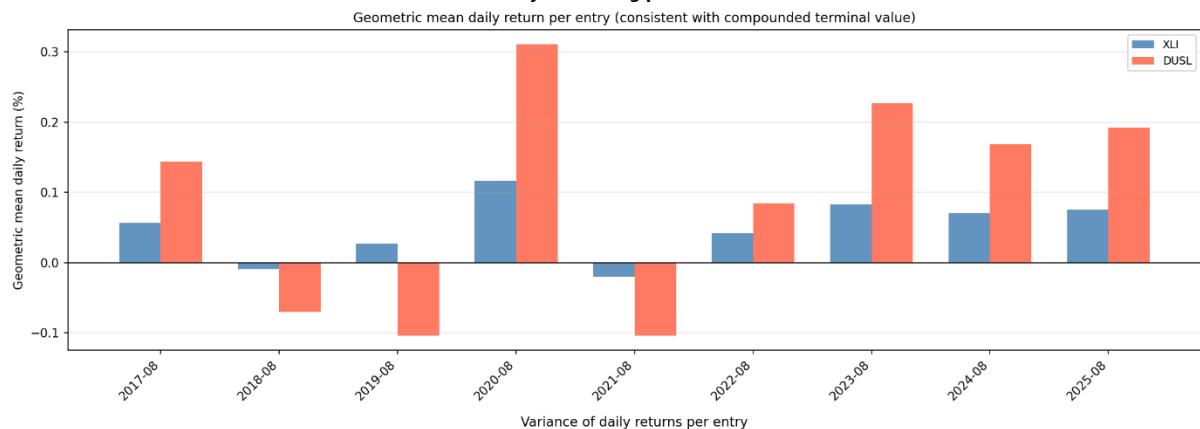




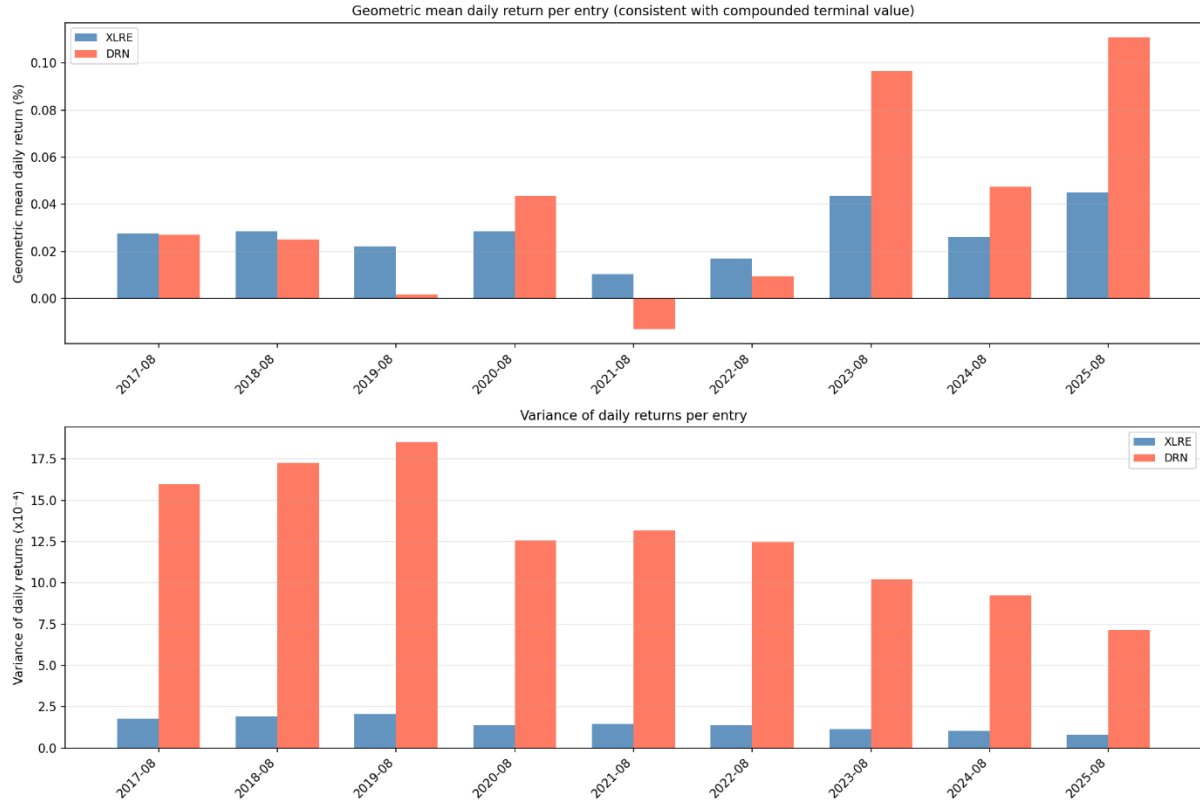
DUSL vs XLI — Mean & Variance of Daily Returns Each entry held until today



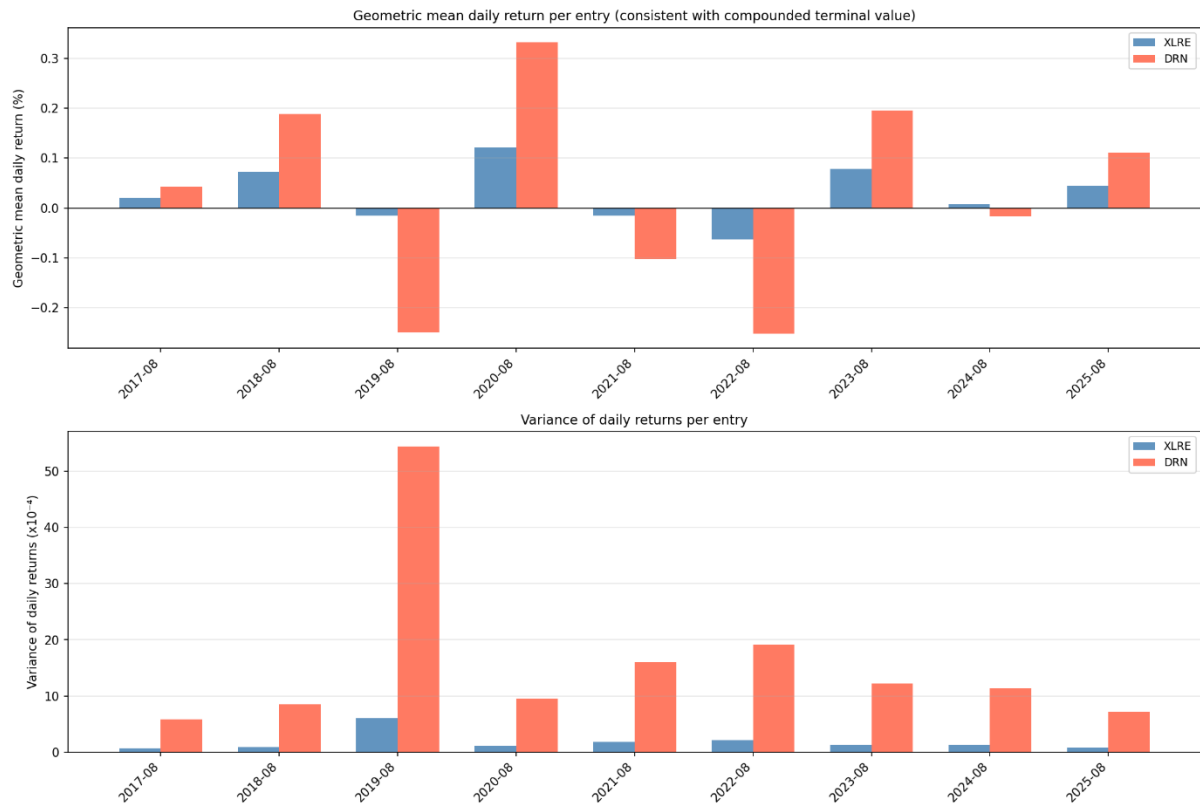
DUSL vs XLI — Mean & Variance of Daily Returns 1-year holding period



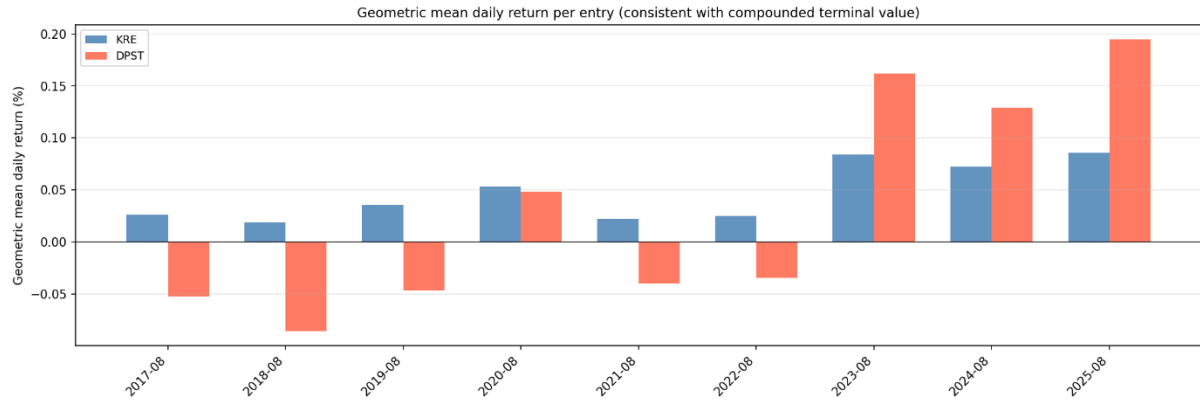
DRN vs XLRE — Mean & Variance of Daily Returns Each entry held until today



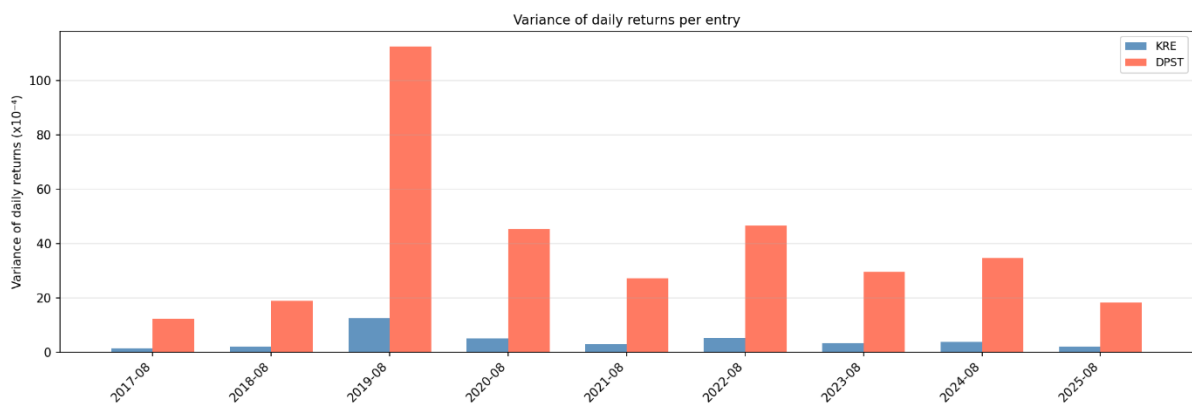
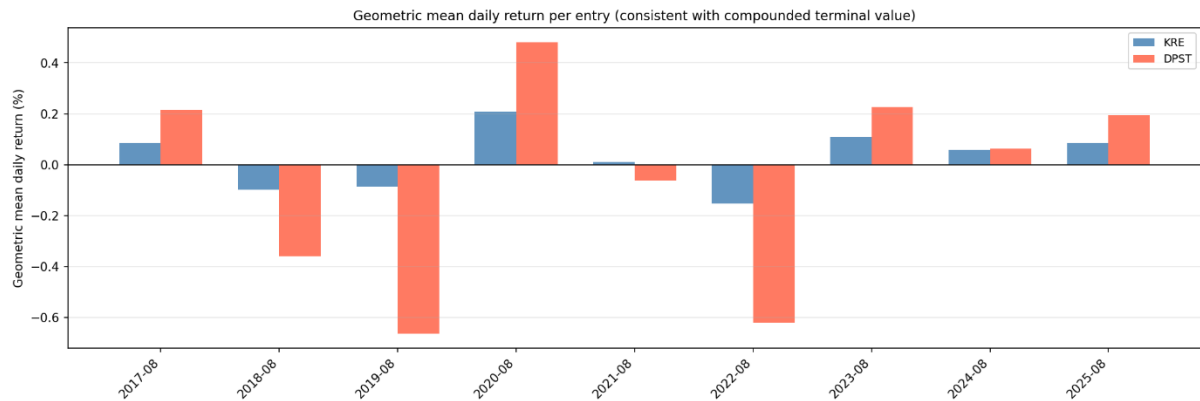
DRN vs XLRE — Mean & Variance of Daily Returns 1-year holding period



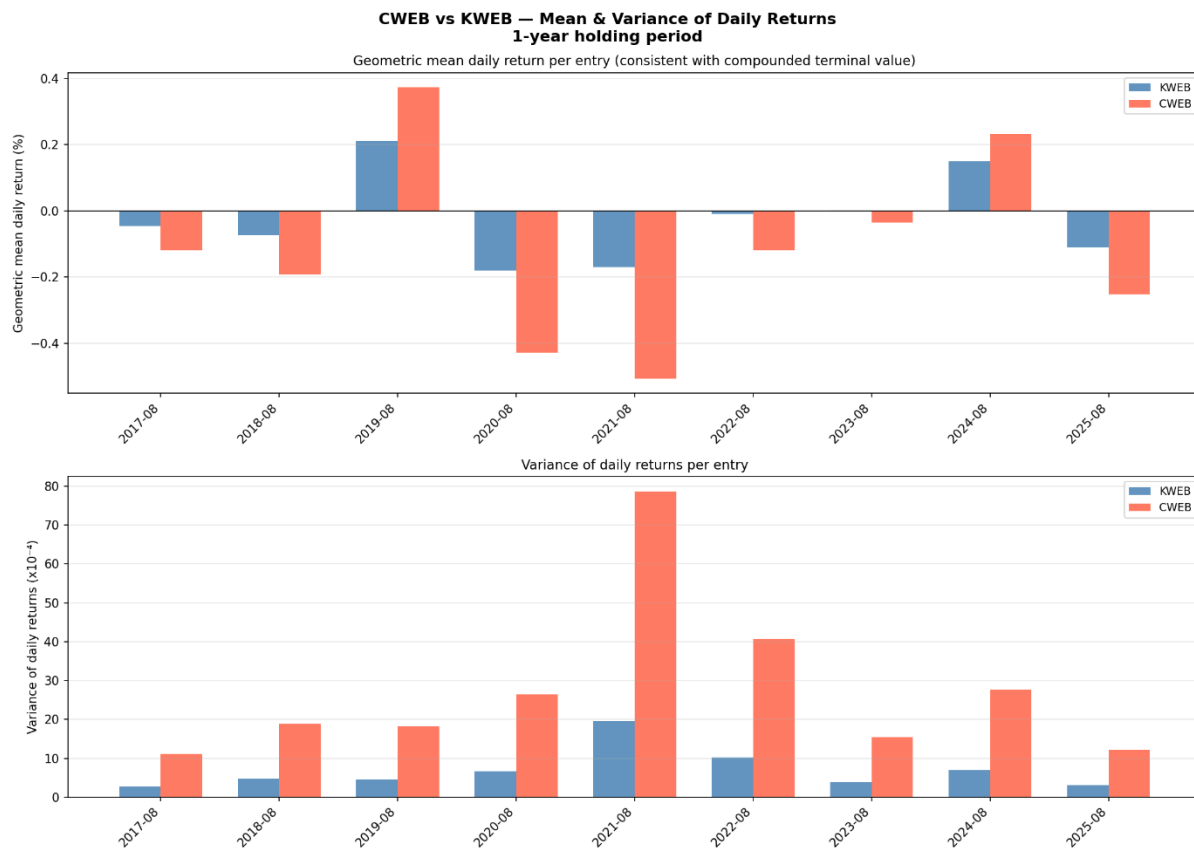
DPST vs KRE — Mean & Variance of Daily Returns Each entry held until today



DPST vs KRE — Mean & Variance of Daily Returns 1-year holding period



Geometric mean daily return per entry (consistent with compounded terminal value)



4.3. Discussion

The observations below are drawn from the full set of results generated in this project, 18 leveraged ETFs matched to 16 distinct benchmarks, each simulated from nine annual entry dates spanning roughly nine years, under two holding rules. The first rule holds each position from its entry date until the present, producing nine entries of decreasing length. The second rule holds each position for a fixed twelve months regardless of entry date, which isolates entry timing from holding-period length. Together these produce 324 entry cases, 162 of them under the fixed one-year rule. All averages reported here are geometric means of daily returns, which are the only averages consistent with the compounded values shown in the graphs.

Butterfly Effect

Across all eighteen leveraged ETFs examined, the charts display the same behaviour, each simulated fund is initialised at \$100 and differs from its counterparts only in the date on which it was created, yet the resulting curves separate widely. Had a fund been created on one date rather than another, its value would differ by a multiple. This separation is not apparent at the outset, in the fixed holding-period charts the curves remain visually indistinguishable for the first several weeks of trading, occupying a narrow band around their common starting value but once established it persists. The effect is visible in every scenario chart, where the portfolio changes greatly between each date of creation of the LETF and the effect that it has on the variance and the mean of the LETFs. This pattern holds regardless of sector, benchmark, or leverage factor, and it appears in funds that ended the period profitably as well as in those that did not. The entry date therefore can affect the outcome considerably, timing the market on these assets is incredibly important as missing the optimal entry by just a year or two can have dire consequences on the performance of our portfolio. The variance charts show that volatility is not a fixed property of a fund but a property of the period in which it is held. Across the nine entry windows the variance of the benchmark changes substantially from one entry to the next, and this holds for every fund examined, in each case the most volatile window carried at least three times the variance of the quietest, and typically closer to six times. An investor selecting an entry date is therefore also selecting, without being able to observe it beforehand, the level of

volatility to which the position will be exposed. For a holder of the benchmark this selection has limited consequence. Variance affects the smoothness of the path more than it affects the destination, and outcomes across quiet and turbulent windows remain within a comparable range. For a holder of the leveraged fund the same selection matters considerably more. The variance of the leveraged position equals the benchmark variance multiplied by the square of the leverage factor, so a difference between entry windows that is modest at 1x becomes a large one at 2x or 3x. Variance also enters the compounded return as a continuing cost rather than a temporary fluctuation, and that cost is incurred on every day the position remains open. The consequence is that entry windows which are broadly interchangeable from the perspective of the benchmark are not interchangeable from the perspective of the leveraged fund, and what separates them is a quantity the investor cannot observe at the moment of entry.

Losses are difficult to reverse

4 of the 18 funds compounded at a negative mean daily return over the study period. Of these, 2 tracked benchmarks that compounded positively, meaning leverage alone reversed the sign of the result. A favourable entry improves the outcome by a bounded amount, whereas an unfavourable entry can require gains of several hundred percent to return to breakeven. The two are not symmetric, which is why entry conditions carry more weight for leveraged products than for their benchmarks.



5. Conclusions

We analyse leveraged ETFs by simulating their creation at several distinct points in time and applying a uniform holding period to each entry. Because every simulated position is initialised with the same capital and differs only in its starting date, any difference in outcome is attributable to the entry alone. On this basis the results show that a change in the date of entry, small relative to the length of the holding period, is sufficient to produce an entirely different outcome. The effect is present in each of the three quantities recorded over the life of the positions. The return on investment separates the entries into a wide range of terminal values from a common starting point. The variance shows that each entry places the position into a different volatility regime, and that this regime persists for the duration of the holding period. The mean daily return shows that the differences are not confined to the shape of the path but appear in the average rate at which the position compounded, so that entries which are broadly similar at the level of the benchmark do not remain similar once leverage is applied.

Two features of the design make this conclusion possible. Fixing the holding period across all entries removes duration as an explanation, so the observed differences cannot be attributed to one position having been held longer than another.

Initialising every position at the same value removes scale, so the resulting curves are directly comparable. What remains once both are controlled is the entry date, and it accounts for the full spread of results.



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